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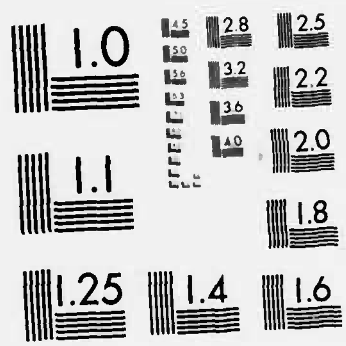

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CALCULATION OF WAVE PACKET TRAJECTORIES AND WAVE HEIGHTS FOR VARIABLE WATER DEPTHS AND CURRENTS

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DEPARTMENT OF OCEANOGRAPHY AND OCEAN ENGINEERING
FLORIDA INSTITUTE OF TECHNOLOGY

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ABSTRACT

The theory and numerical methods are presented for determining the paths and wave heights of gravity water wave packets. Both variable water depths and currents are considered. The wave height is computed accounting for the effects of shoaling, refraction, and energy dissipation. A ray curvature expression is used to determine the wave packet trajectories where the speed of the packet is given by $G = (d\omega/dk) \cos \phi$. The symbol ω denotes the angular frequency, k is the wave number, and ϕ is the difference between the direction of the wave packet and the direction of the wavelets within the packet. The wavelet direction is determined at each point of the wave packet trajectory. The wave packet and ray directions differ when there are currents. The calculations for variations in water depth are greatly simplified by choosing a coordinate system at each ray point in which one axis is aligned parallel with the direction of the gradient of the water depth. In a similar fashion, the calculations involving variations in current are simplified by choosing a coordinate system at each ray point in which an axis is taken parallel with the direction of the gradient of the current magnitude. Example printouts and plots are presented to illustrate the wave prediction method.

ACKNOWLEDGMENTS

We express our appreciation to Sue Downing and Ellen Walker Breeding for their assistance in preparing this report. This research was supported by the Coastal Sciences Program, Office of Naval Research under Contract No. N00014-80-C-0677.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

It has been shown by Breeding (1978, 1981) that a wave packet refracts according to Snell's law with the geometric group velocity G where

$$G = (\partial\omega/\partial k)\cos\phi \tag{1-1}$$

The symbol ω denotes the angular frequency and k is the wave number. The angle ϕ is the difference between the direction of the wave packet and the direction of the wavelets within the packet. The wavelet direction at each point of the wave packet trajectory is determined by Snell's law with phase velocity.

In this work a numerical method is presented for determining the rays of gravity water wave packets. Further, a procedure is developed for computing the wave heights along the paths accounting for the effects of shoaling, refraction, and energy dissipation. Both a variable water depth and current are considered.

There are a number of papers in which numerical methods are presented for calculating and plotting the trajectories of monochromatic rays. Skovgaard, et al (1975) summarize a number of these methods and present one of their own. The numerical methods for calculating and plotting rays which are presented in this work are based on the Wilson (1966) program. However, extensive modifications of the Wilson program were required in order to compute the path of a wave packet. Wilson did not consider wave heights.

In Chapter II equations defining the trajectories of wave packets in a current are presented. An expression is derived for the ray curvature of a wave packet. The Doppler shift in the frequency of the waves due to a current is considered. Properties of the packet ray curvature are discussed for the particular case of parallel water depth contours and no currents. The methods used in computing the wavelet direction are described. The numerical procedure for locating each ray point is discussed. Rules for dealing with reflection points due to water depth variations, which occur when the ray curvature becomes infinite, are established. The condition for the total reflection of waves due to a current is presented.

The shoaling, refraction, and friction coefficients used to compute the wave heights are described. The ray separation equation is discussed. An analytical solution to the equation is presented for the case of parallel wave speed contours. In the program a numerical method is used to solve the ray separation equation. This method is discussed, and the procedure for dealing with reflection points is described. The method for locating caustics and focal points is explained. Near a caustic an approximate solution of the ray separation equation is used in the calculations. This solution is presented. There is a discussion of the wave breaking criterion.

The variations due to water depth and currents are considered separately. The partial derivatives of the wave speeds are related to the corresponding water depth derivatives. The calculations are greatly simplified by making them in a $x'y'$-coordinate system which is chosen so that at each ray point

the positive x'-axis is in the direction of the gradient of the water depth. As a result, the first partial derivatives of the water depth and wave speeds with respect to y'-vanish. Further, there is a simplification in the second partial derivatives involving y'. In a similar manner, a x"y"-coordinate system is used to determine the partial derivatives of the wave speeds and to relate them to the variations in a current. The positive x"-axis is taken in the direction of the gradient of the current speed. This results in a simplification of all the derivatives involving y". The chapter concludes with a summary of the basic equations.

Chapter III contains details of the computer program and a program listing. The program listing contains many comments to explain the operation of the program.

Chapter IV contains details on how to prepare the water depth and current grids. The way to prepare a computer run is explained, and the printed output is described. Both the printout and plots are illustrated. Included are the results for sample data which are provided so that a user can test the program.

The principal notation used in the report is presented following the references.

CHAPTER II THEORY

2.1 Trajectories of Gravity Water Wave Packets. In this section a method is presented for determining the surface trajectories of gravity water wave packets considering both a variable water depth and a variable current.

a. Ray paths

Arthur (1950) considered the trajectories of monochromatic waves in a current. His method is easily extended to determine the trajectories of wave packets in a current. The ray paths are defined by

$$\frac{\partial x}{\partial t} = u_x + G \cos \theta \tag{2-1}$$

$$\frac{\partial y}{\partial t} = u_y + G \sin \theta \tag{2-2}$$

where x,y are the Cartesian coordinates, t is time, and θ is the direction of the wave packet with respect to the positive x-axis. The x- and y-components of the horizontal component of the current velocity are denoted by u_x and u_y where u is the current speed. The geometric group speed of the wave packet relative to the current is G. The ray direction ρ is defined by

$$\tan \rho = \frac{u_y + G \sin \theta}{u_x + G \cos \theta} \tag{2-3}$$

b. Ray curvature for wave packets

The ray curvature κ_v of a ray moving with phase speed v over a variable bottom topography was derived by Munk and Arthur (1952) and Arthur, et al (1952) as

$$K_v = \frac{\partial \gamma}{\partial s_v} = \frac{1}{v}\left(\sin \gamma \frac{\partial v}{\partial x} - \cos \gamma \frac{\partial v}{\partial y}\right) \tag{2-4}$$

where γ is the direction of the monochromatic wave with respect to the positive x-axis and s_v is the arc length along the ray.

The ray curvature κ_G for the trajectory of a wave packet for a variable bottom topography is given by

$$\kappa_G = \frac{d\theta}{ds_G} = \frac{1}{G}\left(\sin\theta \frac{\partial G}{\partial x} - \cos\theta \frac{\partial G}{\partial y}\right) \tag{2-5}$$

where ds_G is an element of arc length along the ray.

The ray curvature expressions are readily extended to include variable currents. For monochromatic rays (Arthur, 1950)

$$\kappa_v = \frac{1}{v + u_k}\left[\sin\gamma\left(\frac{\partial v}{\partial x} + \frac{\partial u_k}{\partial x}\right) - \cos\gamma\left(\frac{\partial v}{\partial y} + \frac{\partial u_k}{\partial y}\right)\right] \tag{2-6}$$

$$u_k = \hat{e}_k \cdot \vec{u} \tag{2-7}$$

where $\hat{e}_k$ is a unit vector in the direction of γ, u_k is the component of the current in the direction of γ, and v is the phase speed relative to the current. For wave packets

$$\kappa_G = \frac{1}{G + u_m}\left[\sin\theta\left(\frac{\partial G}{\partial x} + \frac{\partial u_m}{\partial x}\right) - \cos\theta\left(\frac{\partial G}{\partial y} + \frac{\partial u_m}{\partial y}\right)\right] \tag{2-8}$$

$$u_m = \hat{e}_m \cdot \vec{u} \tag{2-9}$$

where $\hat{e}_m$ is a unit vector in the direction of θ and u_m is the component of the current in the direction of θ.

The geometric group speed is defined (Breeding, 1978)

$$G = U\cos\phi \tag{2-10}$$

where

$$U = \frac{\partial \omega}{\partial k} \tag{2-11}$$

$$\phi = \theta - \gamma \tag{2-12}$$

and U is the conventional (collinear) group speed. The angular frequency $\omega = 2\pi f$, $f = 1/T$ is the frequency, and T is the wave period. These quantities are all defined relative to the current where

$$\omega = \omega_b - \vec{k} \cdot \vec{u} \tag{2-13}$$

and ω_b is the angular frequency in a fixed coordinate system relative to the water bottom. The wave number $k = 2\pi/\lambda$ and λ is the wavelength.

The first partial derivatives of G are determined using Equation (2-10).

$$\frac{\partial G}{\partial x} = \frac{\partial U}{\partial x} \cos\phi - U \sin\phi \frac{\partial \phi}{\partial x} \tag{2-14}$$

$$\frac{\partial G}{\partial y} = \frac{\partial U}{\partial y} \cos\phi - U \sin\phi \frac{\partial \phi}{\partial y} \tag{2-15}$$

Through the use of Equation (2-8) the spatial derivatives of θ are taken to be

$$\frac{\partial \theta}{\partial x} = \frac{\tan\theta}{G + u_m} \left(\frac{\partial G}{\partial x} + \frac{\partial u_m}{\partial x} \right) \tag{2-16}$$

$$\frac{\partial \theta}{\partial y} = \frac{-\cot\theta}{G + u_m} \left(\frac{\partial G}{\partial y} + \frac{\partial u_m}{\partial y} \right) \tag{2-17}$$

In like manner, from Equation (2-6) it is found that

$$\frac{\partial \gamma}{\partial x} = \frac{\tan\gamma}{v + u_{\ell}}\left(\frac{\partial v}{\partial x} + \frac{\partial u_{\ell}}{\partial x}\right) \tag{2-18}$$

$$\frac{\partial \gamma}{\partial y} = \frac{-\cot\gamma}{v + u_{\ell}}\left(\frac{\partial v}{\partial y} + \frac{\partial u_{\ell}}{\partial y}\right) \tag{2-19}$$

When Equations (2-16) and (2-18) are substituted into Equation (2-14) and the terms are rearranged, it is found that

$$\frac{\partial G}{\partial x} = \left(1 + \frac{U \sin\phi \tan\theta}{G + u_m}\right)^{-1} \left\{\frac{\partial U}{\partial x}\cos\phi + U \sin\phi \left[\frac{\tan\gamma}{v + u_{\ell}}\left(\frac{\partial v}{\partial x} + \frac{\partial u_{\ell}}{\partial x}\right) - \frac{\tan\theta}{G + u_m}\frac{\partial u_m}{\partial x}\right]\right\} \tag{2-20}$$

The substitution of Equations (2-17) and (2-19) into Equation (2-15) leads to

$$\frac{\partial G}{\partial y} = \left(1 - \frac{U \sin\phi \cot\theta}{G + u_m}\right)^{-1} \left\{\frac{\partial U}{\partial y}\cos\phi + U \sin\phi \left[\frac{\cot\theta}{G + u_m}\frac{\partial u_m}{\partial y} - \frac{\cot\gamma}{v + u_{\ell}}\left(\frac{\partial v}{\partial y} + \frac{\partial u_{\ell}}{\partial y}\right)\right]\right\} \tag{2-21}$$

In the event there are no currents Equations (2-20) and (2-21) reduce to

$$\frac{\partial G}{\partial x} = \frac{\frac{\partial U}{\partial x}\cos\phi + \frac{U}{v}\sin\phi\tan\gamma\frac{\partial v}{\partial x}}{1 + \tan\phi\tan\theta}, \quad u = 0 \tag{2-22}$$

$$\frac{\partial G}{\partial y} = \frac{\frac{\partial U}{\partial y}\cos\phi - \frac{U}{v}\sin\phi\cot\gamma\frac{\partial v}{\partial y}}{1 - \tan\phi\cot\theta}, \quad u = 0 \tag{2-23}$$

In the computations it is convenient to separate out the variations due to water depth and the variations in frequency due to a variable current. As a result, if only one of the types of variation is being considered the equations associated with the other type of variation can be ignored. The

ray curvature for the wave packet is expressed as

$$K_G = \frac{1}{G + u_m}\left[\left\{\sin\theta \frac{\partial G}{\partial x} - \cos\theta \frac{\partial G}{\partial y}\right\}_h + \left\{\sin\theta\left(\frac{\partial G}{\partial x} + \frac{\partial u_m}{\partial x}\right) - \cos\theta\left(\frac{\partial G}{\partial y} + \frac{\partial u_m}{\partial y}\right)\right\}_\omega\right] \tag{2-24}$$

where the subscript h denotes terms in which a variation in water depth is considered, and the subscript ω refers to terms in which changes in current are accounted for.

Since the wave packet and the ray travel with different velocities, the incremental distances by which they advance for a given time interval are different. The wave packet incremental distance must extend to the same wave speed contour reached by the ray. This is illustrated in Figure (2-1) where the wave speed contours are assumed to be locally parallel. From the figure, it is seen that

$$ds_G = \frac{\cos\rho}{\cos\theta}\, ds_{ray} \tag{2-25}$$

c. Properties of the wave packet ray curvature

The ray curvature of a wave packet has been examined (Breeding, 1981) for the case of parallel water depth contours and no currents. If the water depth contours are parallel to the y-axis the packet ray curvature becomes

$$K_G = \frac{\dfrac{1}{U}\dfrac{\partial U}{\partial x} + \dfrac{\tan\phi \tan\gamma}{v}\dfrac{\partial v}{\partial x}}{\csc\theta + \tan\phi \sec\theta} \tag{2-26}$$

From this expression it is found that refraction causes a wave packet trajectory to become directed either parallel or perpendicular to the water depth contours. In either case the packet ray curvature will vanish. For wave packets propogating toward deep water, if the initial direction exceeds a critical angle total reflection occurs. At the reflection point the wavelet direction becomes parallel to the wave speed contours, the wave packet direction becomes perpendicular to the contours, the geometric group velocity goes to zero, and the packet ray curvature becomes infinite.

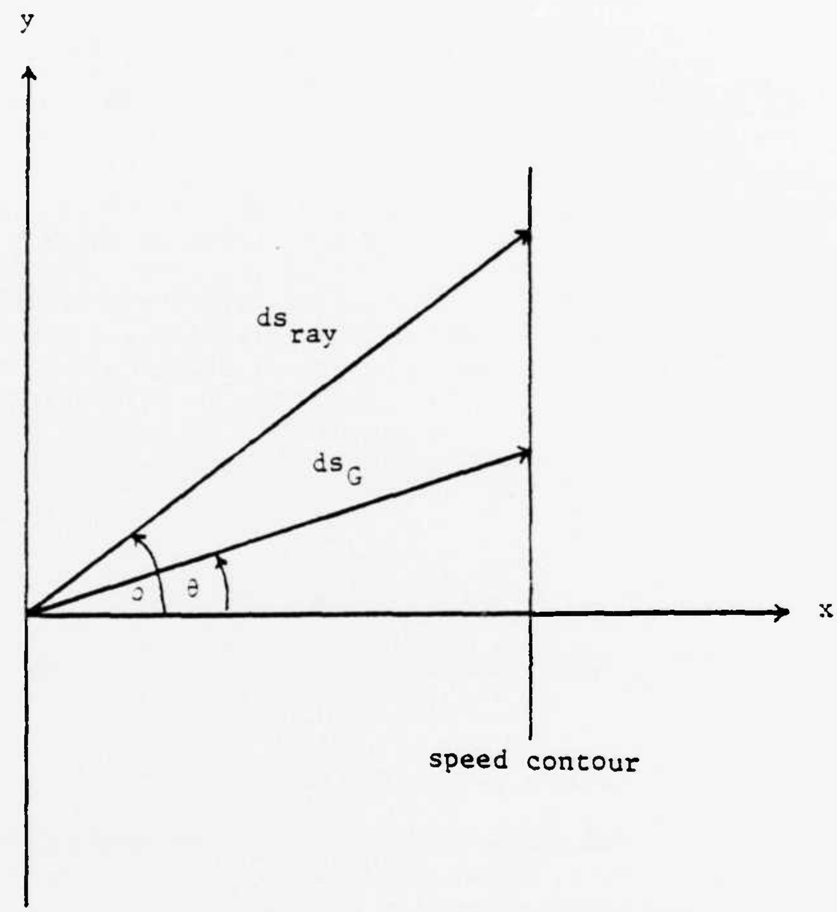


Figure (2-1). RELATIONSHIP BETWEEN THE RAY AND WAVE PACKET INCREMENTAL DISTANCES

d. Wavelet direction

Both the wave packet and wavelet directions must be computed in determining each point of a ray path. When variations occur in both the water depth and a current the change in the wavelet direction is computed using the wavelet ray curvature expression. It is found that the change in wavelet direction is given by

$$\Delta\gamma = \left[\left\{ \cos\rho \left(\tan\gamma \frac{\partial v}{\partial x} - \frac{\partial v}{\partial y} \right) \right\}_h + \left\{ \cos\rho \left(\tan\gamma \left(\frac{\partial v}{\partial x} + \frac{\partial u_R}{\partial x} \right) - \left(\frac{\partial v}{\partial y} + \frac{\partial u_R}{\partial y} \right) \right) \right\}_\omega \right] \frac{G_R \Delta t}{v + u_R} \tag{2-27}$$

where G_R is the ray speed and Δt is the time step interval between ray points. The variations due to water depth and current are considered separately. In Equation (2-27) the difference in the incremental distances which rays and wavelets advance for a given Δt has been taken into account.

The wavelet direction can be calculated using Snell's law with phase velocity. This approach has been found to be both more accurate and to require fewer calculations in locating the next ray point than when the ray curvature expression is used to compute the wavelet direction. Accordingly, if variations are to be considered only in the water depth or a current, but not both, Snell's law is used to compute the wavelet direction.

In order to use Snell's law, the incident wavelet angle is defined with respect to the normal to the wave speed contours which is extended in the direction of increasing wave speed. The wave speed contours are assumed to be locally parallel about the ray point. In the computations a number of rules are employed where the subscripts n and (n + 1) refer to consecutive points of a ray and n is a positive integer. The first step, if necessary, is to successively add or subtract 360° from the incident wavelet angle until it is within the range $0 \leq \gamma_n < 360°$. Then when Snell's law is given by

$$\gamma_{n+1}^{*} = \arcsin\left(\frac{v_{n+1}}{v_n} \sin\gamma_n \right) \tag{2-28}$$

where $-90° \leq \gamma_{n+1}^{*} \leq 90°$, the angle γ_{n+1} is defined by the following scheme.

$$\gamma_{n+1} = \begin{cases} \gamma_{n+1}^{*}, & \gamma_n \leq 90° \\ 180° - \gamma_{n+1}^{*}, & 90° < \gamma_n \leq 270° \\ 360° + \gamma_{n+1}^{*}, & \gamma_n > 270° \end{cases} \tag{2-29}$$

e. Determining the path

Successive points of a ray are found by iteration using the ray curvature expression for a wave packet. The wavelet direction is determined at each point along the ray. The coordinates of each ray point are defined by

$$X_{m+1} = X_m + \Delta X \tag{2-30}$$

$$y_{m+1} = y_m + \Delta y \tag{2-31}$$

where

$$\Delta X = (G \cos\bar{\theta} + u \cos\epsilon)\frac{\Delta t}{GRID} \tag{2-32}$$

$$\Delta y = (G \sin\bar{\theta} + u \sin\epsilon)\frac{\Delta t}{GRID} \tag{2-33}$$

and where ϵ is the direction of the current with respect to the positive x-axis. Further

$$\bar{\theta} = \frac{1}{2}(\theta_m + \theta_{m+1}) \tag{2-34}$$

$$\theta_{m+1} = \theta_m + \Delta\theta \tag{2-35}$$

$$\Delta\theta = \frac{1}{2}\left[(K_G)_m + (K_G)_{m+1}\right] D_m \tag{2-36}$$

$$D_n = \frac{G_R(\Delta t)}{GRID} \qquad (2\text{-}37)$$

where D_n is the incremental distance in grid units between the points n and (n ± 1) of a ray, GRID is the grid unit distance in units consistent with G, and Δt is the time step.

f. Reflection points for water depths

Reflection points due to variations in water depth require special consideration. The waves are assumed to reflect if any of three criteria are satisfied. The first test for reflection is based on Snell's law with phase velocity. Reflection occurs if

$$\left| \frac{v_{n+1}}{v_n} \sin \gamma \right| > 1 \qquad (2\text{-}38)$$

The wavelet angle is defined with respect to the normal to the wave speed contour which is extended in the direction of increasing water depth.

As a reflection point is approached the ray curvature changes so quickly that calculations of the ray curvature by iteration may cease to converge. If convergence fails reflection is assumed if the following conditions are met.

$$\frac{v_{n+1}}{v_n} > 1$$
$$|\tan \gamma| > \tan 80^\circ \qquad (2\text{-}39)$$

The first condition requires that the phase speed increases between the last two ray points, and the second condition requires that the wavelet direction be consistent with total reflection.

A third criterion is used to specify reflection points in order to maintain accuracy in calculating the ray trajectory. Very near a reflection point, due to the rapidly changing ray curvature, the estimates of ray points can become erratic. Therefore, reflection is assumed if the following conditions are met

$$\frac{v_{n+1}}{v_n} > 1$$
$$|\tan \gamma| > \tan 89.5^\circ \qquad (2\text{-}40)$$
$$|\tan \theta| < \tan 75^\circ$$

When a reflection point is determined on the basis of either the second or third criterion, it is advisable to examine the printout to determine if the ray particulars are consistent with total reflection. The values of the packet ray curvature, the wavelet direction, the packet direction, and the geometric group velocity should exhibit the behavior described in Section (2.1), c.

When a reflection occurs there is an option either to halt the ray trajectory or to continue it as a reflection. When the reflection option is chosen, the reflection angles (defined with respect to the normal to the water depth contour) are determined by the relations

$$\gamma_r = -\gamma_i + 180° \tag{2-41}$$

$$\theta_r = -\theta_i + 180° \tag{2-42}$$

$$\rho_r = -\rho_i + 180° \tag{2-43}$$

The subscript r denotes the reflection angles, and the subscript i signifies the angles at the last ray point reached prior to where the reflection criterion is satisfied.

The ray curvature calculations are more likely to converge and the accuracy of the path is increased if there is a restriction on how much the ray direction can change between successive ray points. Accordingly, if γ is within 10° of a direction for which the ray curvature is infinite, and θ is within 75° of being perpendicular to the water depth contour, the time step between ray points is successively halved, as necessary, until $|\Delta\theta|$ is less than 1°. In the event it is necessary to reduce the time step to less than 0.5 seconds the ray is stopped.

g. Total reflection for currents

Total reflection due to variations in a current is determined on the basis of Snell's law for the wavelets. Reflection occurs if

$$\left| \frac{(v + u_z)_{m+1}}{(v + u_z)_m} \sin \gamma \right| > 1 \tag{2-44}$$

where the wavelet angle is defined with respect to the normal to the wave speed contour which is extended in the direction of increasing current speed.

2.2 Wave Height Calculations. Modification of the wave height due to refraction, shoaling, and energy dissipation is considered. The wave height H increases when adjacent rays converge and it decreases when the rays diverge; this effect is accounted for by the refraction coefficient K_R. The shoaling coefficient K_S accounts for the change in H due to the variation in the propagation speed of the wave packet. The loss in energy due to wave motion at the sea bottom is determined by the friction coefficient K_F.

a. Without energy dissipation

The average rate of energy transmission F can be defined

$$F = E \ell G_T \tag{2-45}$$

where $G_T = G + u_m$, E is the energy per unit area, and ℓ is the perpendicular distance between rays. Note that G_T is the speed of the advected group front. The energy is assumed to be conserved along the ray. Therefore

$$F_{j+1} = F_j \tag{2-46}$$

where j and (j + 1) denote consecutive ray points. It is further assumed that E is proportional to H^2. Accordingly, it follows from Equations (2-45) and (2-46) that

$$H_{j+1} = (K_S)_{j+1} (K_R)_{j+1} H_j \tag{2-47}$$

where $(K_S)_{j+1}$ and $(K_R)_{j+1}$ are the shoaling and refraction coefficients, respectively, between points j and (j + 1).

$$(K_S)_{j+1} = \left(\frac{(G_T)_j}{(G_T)_{j+1}} \right)^{\frac{1}{2}} \tag{2-48}$$

$$(K_R)_{j+1} = \left(\frac{\ell_j}{\ell_{j+1}} \right)^{\frac{1}{2}} \tag{2-49}$$

If H_0 is the initial wave height, then the wave height at the n-th point is

$$H_n = K_S K_R H_0 \tag{2-50}$$

where

$$K_S = (K_S)_1 (K_S)_2 \cdots (K_S)_n = \left(\frac{(G_T)_0}{(G_T)_n} \right)^{\frac{1}{2}} \tag{2-51}$$

$$K_R = (K_R)_1 (K_R)_2 \cdots (K_R)_n = \left(\frac{\ell_0}{\ell_n} \right)^{\frac{1}{2}} \tag{2-52}$$

b. With energy dissipation

To account for energy losses, Equation (2-46) can be restated

$$F_{j+1} = (K_F)_{j+1}^{2} F_j \tag{2-53}$$

where $(K_F)_{j+1}$ is the friction coefficient between the points j and (j + 1). As a result, the relationship between the wave heights at consecutive ray points can be expressed by

$$H_{j+1} = (K_S)_{j+1} (K_R)_{j+1} (K_F)_{j+1} H_j \tag{2-54}$$

In terms of the initial wave height, the wave height at the n-th point is given by

$$H_n = K_S K_R K_F H_0 \tag{2-55}$$

where

$$K_F = (K_F)_1 (K_F)_2 \cdots (K_F)_m = (K_F)_m (K_F)_m \qquad (2\text{-}56)$$

2.3 Refraction Coefficient. In computing K_R it is convenient to define

$$\beta = \frac{\ell_m}{\ell_0} \qquad (2\text{-}57)$$

where β is called the ray separation factor. Equation (2-52) for the refraction coefficient becomes

$$K_R = |\beta|^{-\frac{1}{2}} \qquad (2\text{-}58)$$

a. Ray separation equation

In considering the refraction of monochromatic waves, Munk and Arthur (1952) have shown that β can be determined from a second-order differential equation called the ray separation equation. The equation can be stated

$$\frac{d^2\beta}{dt^2} + p\frac{d\beta}{dt} + q\beta = 0 \qquad (2\text{-}59)$$

where t is time. For monochromatic waves with no currents

$$p = -2\left(\cos\gamma \frac{\partial v}{\partial x} + \sin\gamma \frac{\partial v}{\partial y}\right) \qquad (2\text{-}60)$$

$$q = v\left(\sin^2\gamma\,\frac{\partial^2 v}{\partial x^2} - 2\sin\gamma\cos\gamma\,\frac{\partial^2 v}{\partial x\partial y} + \cos^2\gamma\,\frac{\partial^2 v}{\partial y^2}\right) \qquad (2\text{-}61)$$

For a wave packet trajectory with currents

$$p = -2\left[\cos\theta\left(\frac{\partial G}{\partial x} + \frac{\partial U_m}{\partial x}\right) + \sin\theta\left(\frac{\partial G}{\partial y} + \frac{\partial U_m}{\partial y}\right)\right] \qquad (2\text{-}62)$$

$$q = G_T\left[\sin^2\theta\left(\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 U_m}{\partial x^2}\right) - 2\sin\theta\cos\theta\left(\frac{\partial^2 G}{\partial x\partial y} + \frac{\partial^2 U_m}{\partial x\partial y}\right) + \cos^2\theta\left(\frac{\partial^2 G}{\partial y^2} + \frac{\partial^2 U_m}{\partial y^2}\right)\right] \qquad (2\text{-}63)$$

As in the case of computing the ray curvature (Section (2.1), b), it is convenient to separate out the variations due to water depth and a current.

$$p = -2\left[\left\{\cos\theta\,\frac{\partial G}{\partial x} + \sin\theta\,\frac{\partial G}{\partial y}\right\}_h + \left\{\cos\theta\left(\frac{\partial G}{\partial x} + \frac{\partial U_m}{\partial x}\right) + \sin\theta\left(\frac{\partial G}{\partial y} + \frac{\partial U_m}{\partial y}\right)\right\}_\omega\right] \qquad (2\text{-}64)$$

$$q = G_T\left[\left\{\sin^2\theta\,\frac{\partial^2 G}{\partial x^2} - 2\sin\theta\cos\theta\,\frac{\partial^2 G}{\partial x\partial y} + \cos^2\theta\,\frac{\partial^2 G}{\partial y^2}\right\}_h + \left\{\sin^2\theta\left(\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 U_m}{\partial x^2}\right) - 2\sin\theta\cos\theta\left(\frac{\partial^2 G}{\partial x\partial y} + \frac{\partial^2 U_m}{\partial x\partial y}\right) + \cos^2\theta\left(\frac{\partial^2 G}{\partial y^2} + \frac{\partial^2 U_m}{\partial y^2}\right)\right\}_\omega\right] \qquad (2\text{-}65)$$

b. Solution for parallel water depth contours

There is a simple solution to Equations (2-59), (2-62), and (2-63) when the water depth contours and current contours are mutually parallel. Then, when the xy-coordinate system is chosen with the x-axis perpendicular to the contours, it can be shown that

$$\beta = \frac{\cos \theta_n}{\cos \theta_o} \tag{2-66}$$

where the subscript o denotes the initial value and the subscript n depicts the value at the n-th ray point. The time derivative of Equation (2-66) can be expressed

$$\frac{d\beta}{dt} = - \beta \sin \theta \tan \theta \left(\frac{\partial G}{\partial X} + \frac{\partial U_m}{\partial X} \right) \tag{2-67}$$

c. Numerical solutions of the ray separation equation

Several numerical methods can be used to solve the ray separation equation. An easy to use fourth order finite difference solution to Equation (2-59) is the Fox method (Salvadori and Baron, 1961). However, this method has the disadvantage that the time step must be constant between successive ray points. When p and q do not change much between ray points, the general solution of a homogeneous second-order differential equation with constant coefficients (Wylie, 1951) can be used to solve the ray separation equation. This solution has 3 cases depending on the value of $(p^2 - 4q)$. The value of β at each new ray point is found using the values of p and q at the last point. There is usually little difference between the results obtained by this method and the Fox method.

A numerical method which does not require a constant time step and which better accounts for the variation of p and q along a ray is the Runge-Kutta method. This method is stable. It is also self-starting, i.e., values at the previous point are used to find values at the next point (Romanelli, 1960). For these reasons, the Runge-Kutta method was selected for the solution of the ray separation equation.

In order to use the Runge-Kutta method, Equation (2-59) is reduced to a system of first-order equations.

$$\frac{d\beta}{dt} = u_N \tag{2-68}$$

$$\frac{du_N}{dt} = -(p\,u_N + q\beta) \tag{2-69}$$

Both fourth and fifth order solutions of β are obtained. The initial conditions are the values of β and $d\beta/dt$ at the first ray point. The latter is estimated using Equation (2-67). The solutions require the values of (p_n, q_n) at the n-th ray point and the values (p_{n+1}, q_{n+1}) at the (n+1)-th ray point. Further, the values of (p_1, q_1), (p_2, q_2), (p_3, q_3), (p_4, q_4), and (p_5, q_5) are needed along the ray at points intermediate to the ray points. They are determined, respectively, at time intervals of $(\Delta t)/3$, $(\Delta t)/4$, 0.45573725 (Δt), $2(\Delta t)/3$, and 0.8 (Δt) beyond the n-th ray point where Δt is the time step in the calculations.

A fourth order Runge-Kutta method with a minimum truncation error bound is given by Ralston (1962). The solution for the ray separation equation becomes

$$\begin{aligned}\beta_{n+1} = \beta_n &+ 0.17476028K_1 - 0.55148066K_2 \\ &+ 1.20553560K_3 + 0.17118478K_4\end{aligned} \tag{2-70}$$

$$\begin{aligned}\left(\frac{d\beta}{dt}\right)_{n+1} = \left(\frac{d\beta}{dt}\right)_n &+ 0.17476028L_1 - 0.55148066L_2 \\ &+ 1.20553560L_3 + 0.17118478L_4\end{aligned} \tag{2-71}$$

where

$$K_1 = (\Delta t)\left(\frac{d\beta}{dt}\right)_n \tag{2-72}$$

$$L_1 = -(\Delta t)\left[p_n\left(\frac{d\beta}{dt}\right)_n + q_n\beta_n\right] \tag{2-73}$$

$$K_2 = (\Delta t)\left[\left(\frac{d\beta}{dt}\right)_n + 0.4L_1\right] \tag{2-74}$$

$$L_2 = -(\Delta t)\left[p_2\left(\left(\frac{d\beta}{dt}\right)_n + 0.4L_1\right) + q_2(\beta_n + 0.4K_1)\right] \tag{2-75}$$

$$K_3 = (\Delta t)\left[\left(\frac{d\beta}{dt}\right)_m + 0.29697761 L_1 + 0.15875964 L_2\right] \tag{2-76}$$

$$L_3 = -(\Delta t)\left[p_3\left(\left(\frac{d\beta}{dt}\right)_m + 0.29697761 L_1 + 0.15875964 L_2\right)\right.$$
$$\left. + q_3(\beta_m + 0.29697761 K_1 + 0.15875964 K_2)\right] \tag{2-77}$$

$$K_4 = (\Delta t)\left[\left(\frac{d\beta}{dt}\right)_m + 0.21810040 L_1 - 3.05096516 L_2\right.$$
$$\left. + 3.83286476 L_3\right] \tag{2-78}$$

$$L_4 = -(\Delta t)\left[p_{m+1}\left(\left(\frac{d\beta}{dt}\right)_m + 0.21810040 L_1 - 3.05096516 L_2\right.\right.$$
$$\left. + 3.83286476 L_3\right) + q_{m+1}(\beta_m + 0.21810040 K_1 \tag{2-79}$$
$$\left. - 3.05096516 K_2 + 3.83286476 K_3)\right]$$

A disadvantage of the Runge-Kutta method is that there is no simple means for estimating the truncation error (Milne, 1953). One procedure for controlling the error is to compute both fourth and fifth order solutions of β and to adjust the time step so that the two estimates differ by less than an arbitrary amount.

A fifth order Runge-Kutta method is given by Milne (1953). The fifth order solutions for β and $d\beta/dt$ are

$$\beta_{m+1}^{(5)} = \beta_m + \frac{1}{192}(23K_1 + 125K_6 - 81K_8 + 125K_9) \tag{2-80}$$

$$\left(\frac{d\beta}{dt}\right)_{m+1}^{(5)} = \left(\frac{d\beta}{dt}\right)_m + \frac{1}{192}(23L_1 + 125L_6 - 81L_8 + 125L_9) \tag{2-81}$$

where

$$K_5 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{L_1}{3}\right] \tag{2-82}$$

$$L_5 = -(\Delta t)\left[p_1\left(\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{L_1}{3}\right) + q_1\left(\beta_m + \frac{K_1}{3}\right)\right] \tag{2-83}$$

$$K_6 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{6L_5 + 4L_1}{25}\right] \tag{2-84}$$

$$L_6 = -(\Delta t)\left[p_2\left(\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{6L_5 + 4L_1}{25}\right) + q_2\left(\beta_m + \frac{6K_5 + 4K_1}{25}\right)\right] \tag{2-85}$$

$$K_7 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{15L_6 - 12L_5 + L_1}{4}\right] \tag{2-86}$$

$$\begin{aligned} L_7 = -(\Delta t)\Bigg[& p_{m+1}\left(\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{15L_6 - 12L_5 + L_1}{4}\right) \\ & + q_{m+1}\left(\beta_m + \frac{15K_6 - 12K_5 + K_1}{4}\right)\Bigg] \end{aligned} \tag{2-87}$$

$$K_8 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{8L_7 - 50L_6 + 90L_5 + 6L_1}{81}\right] \tag{2-88}$$

$$\begin{aligned} L_8 = -(\Delta t)\Bigg[& p_4\left(\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{8L_7 - 50L_6 + 90L_5 + 6L_1}{81}\right) \\ & + q_4\left(\beta_m + \frac{8K_7 - 50K_6 + 90K_5 + 6K_1}{81}\right)\Bigg] \end{aligned} \tag{2-89}$$

$$K_9 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{8L_7 + 10L_6 + 36L_5 + 6L_1}{75}\right] \tag{2-90}$$

$$L_q = -(\Delta t)\left[\gamma_5\left(\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{8L_7 + 10L_6 + 36L_5 + 6L_1}{75}\right)\right.$$

$$\left. + q_{\gamma 5}\left(\beta_m + \frac{8K_7 + 10K_6 + 36K_5 + 6K_1}{75}\right)\right] \tag{2-91}$$

The difference between the fourth and fifth order solutions of β and dβ/dt are

$$\epsilon_\beta = \beta_{m+1} - \beta_{m+1}^{(5)} \tag{2-92}$$

$$\epsilon_{\beta t} = \left(\frac{\partial\beta}{\partial t}\right)_{m+1} - \left(\frac{\partial\beta}{\partial t}\right)_{m+1}^{(5)} \tag{2-93}$$

In the calculations, with the exception of when near reflection points, both $|\varepsilon_\beta|$ and $|\varepsilon_{\beta t}|$ are monitored. If either is greater than or equal to an arbitrary constant (determined as an input parameter) the time step is halved, the corresponding (n + 1)-th ray point is found, and the β and dβ/dt calculations are repeated. This process continues, as necessary, until both $|\varepsilon_\beta|$ and $|\varepsilon_{\beta t}|$ are less than the arbitrary constant. If the time step is reduced to less than 0.5 seconds the ray is stopped.

d. Reflection points

The numerical solutions of the ray separation equation have not proved satisfactory when leaving a reflection point. This is possibly due to the rapid changes in p which becomes infinite at the reflection point. However, as the reflection point is approached the value of β approaches a constant. Therefore, if the reflection option is chosen, the value of β is held constant in the calculations if the wavelet direction is within 10° of being parallel to the water depth contours. After passing the reflection point, Equation (2-67) is used to estimate dβ/dt for restarting the Runge-Kutta calculations. If the reflection option is not chosen the calculations of β continue up to the reflection point. As a result, by choosing this option it is possible to determine the value of β at a reflection point.

e. Caustics and focal points

The value of β is monitored along a ray. If the value of β becomes less than 0.0001 (K_R greater than 100) it is assumed that a focal point or caustic has been located. In this case the ray is stopped.

Near a caustic the ray separation equation is given approximately by

$$\frac{d^2\beta}{dt^2} + q\beta = 0 \tag{2-94}$$

In order to improve calculations near a caustic, if variations are considered in only water depth or currents, but not both, Equation (2-94) is used in computing β. For negative values of q the solution to this equation is

$$\beta = e^{-\sqrt{|q|}\,t} \tag{2-95}$$

and

$$\frac{d\beta}{dt} = -\sqrt{|q|}\,\beta \tag{2-96}$$

If the packet direction is within 5° of being parallel to the wave speed contours Equation (2-96) is used to evaluate $d\beta/dt$.

Very small or large values of K_R should be viewed with caution. The values may be incorrect due to numerical inaccuracies in the calculations. This can be checked by examining the behavior of adjacent rays on a plot. For large K_R values the rays should converge closely and for small values of K_R the rays should widely diverge. It is probably best to view K_R values less than 0.2 and greater than 5 in a qualitative sense.

2.4 Friction Coefficient. Energy dissipation of the waves due to bottom friction is considered. The friction coefficient is determined using a method based on the theory of Putnam and Johnson (1949) and Bretschneider and Reid (1954). Other energy dissipation methods can be substituted if desired.

In this work the friction factor c_f is defined following Jonsson (1966)

$$\tau = \frac{1}{2} c_f \rho_f u_M^2 \tag{2-97}$$

where τ is the tangential stress per unit area at the bottom, ρ_f is the density of the fluid, and u_M is the maximum velocity of the fluid at the bottom. The definition for τ given by Putnam and Johnson (1949) does not contain the factor ½. When Equation (2-97) is used the friction coefficient becomes

$$K_F = \frac{(K_F)_m}{F(K_F)_m(\Delta s_G)_{m+1} + 1} \tag{2-98}$$

where $(K_f)_m$ is defined by Equation (2-56), $(\Delta s_G)_{n+1}$ is the incremental distance between the ray points n and (n + 1), and

$$F = \left(\frac{8\pi^2}{3g}\right)\left(\frac{c_f H_o}{U_o}\right)\left(\frac{K_s}{T \sinh kh}\right)^3 \tag{2-99}$$

2.5 Wave Breaking Criterion. In the program there is an option to determine if the waves break. When this option is chosen, the waves are assumed to break when the following relation is satisfied.

$$\frac{H}{\lambda} > \frac{1}{7} \tanh kh \tag{2-100}$$

2.6 Spatial Derivatives of G, U, v, and h for Water Depths. In this section relations are presented for connecting the partial derivatives of G, U, v, and the water depth h. Only variations in water depth are considered. A current is assumed to be either constant or not to exist. The spatial derivatives are determined in a coordinate system where the first order y-partial derivatives vanish and the second order y-derivatives are reduced to simplified expressions. As a result, there is a reduction in the number of calculations which would otherwise be required.

a. Determination of h and its partial derivatives

For each ray point the water depth h is interpolated from a quadratic surface equation which is fitted to the water depths at 12 grid points as illustrated in Figure (2-2). The use of a quadratic surface makes it possible to evaluate second derivatives which are required in calculating the wave height. The surface is approximated by the general quadratic equation (Dobson, 1967)

$$h = E_1 + E_2 x + E_3 y + E_4 x^2 + E_5 xy + E_6 y^2 \tag{2-101}$$

where the coefficients E are determined by fitting the equation by the method of least squares to the 12 water depth values. The partial derivatives of h are readily determined from Equation (2-101).

$$\frac{\partial h}{\partial x} = E_2 + 2E_4 x + E_5 y \tag{2-102}$$

$$\frac{\partial h}{\partial y} = E_3 + E_5 x + 2E_6 y \tag{2-103}$$

$$\frac{\partial^2 h}{\partial x^2} = 2E_4 \tag{2-104}$$

$$\frac{\partial^2 h}{\partial x \partial y} = E_5 \tag{2-105}$$

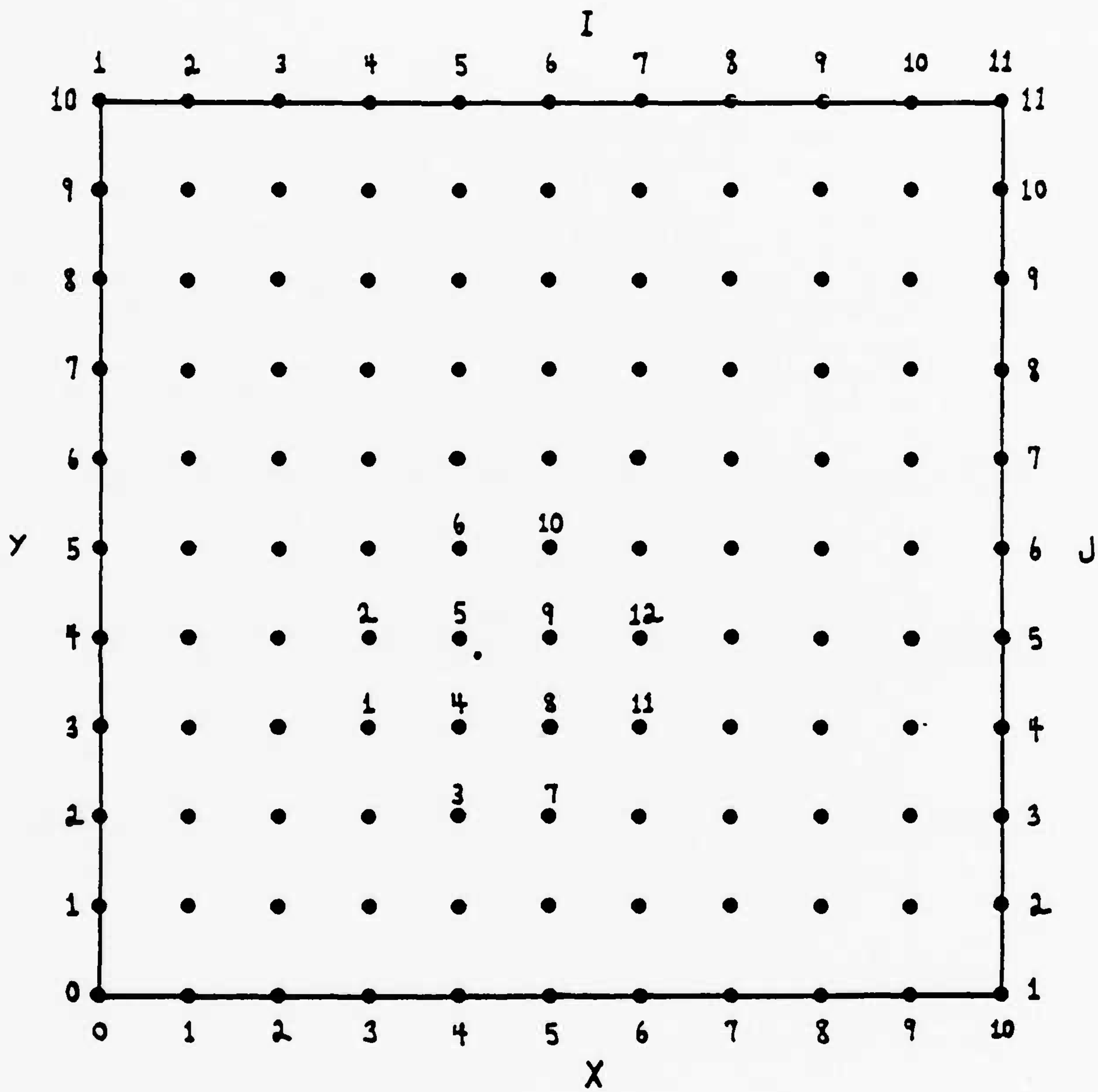


Figure (2-2). SELECTION OF WATER DEPTHS ABOUT RAY POINT (X,Y) = (4.2, 3.8)

$$\frac{\partial^2 h}{\partial y^2} = 2E_6 \qquad (2\text{-}106)$$

b. Rotation of axes to make computations

At each point of a wave packet trajectory the calculations are made in a x'y'-coordinate system where the x'-axis is taken in the direction of the gradient of the water depth. The particulars of a trajectory are tabulated in a xy-coordinate system which retains a fixed orientation with respect to the water depth grid. The relationships between these coordinate systems and a specific ray point for a set of nonparallel water depth contours are shown in Figure (2-3). Equations relating these coordinate systems are given by

$$x' = x \cos\alpha + y \sin\alpha \qquad (2\text{-}107)$$

$$y' = -x \sin\alpha + y \cos\alpha \qquad (2\text{-}108)$$

$$\tan\alpha = \frac{\partial h}{\partial y} \Big/ \frac{\partial h}{\partial x} \qquad (2\text{-}109)$$

where α is the angle by which the x'-axis is rotated with respect to the x-axis and h is the water depth. Note that

$$\rho' = \rho - \alpha \qquad (2\text{-}110)$$

$$\theta' = \theta - \alpha \qquad (2\text{-}111)$$

$$\gamma' = \gamma - \alpha \qquad (2\text{-}112)$$

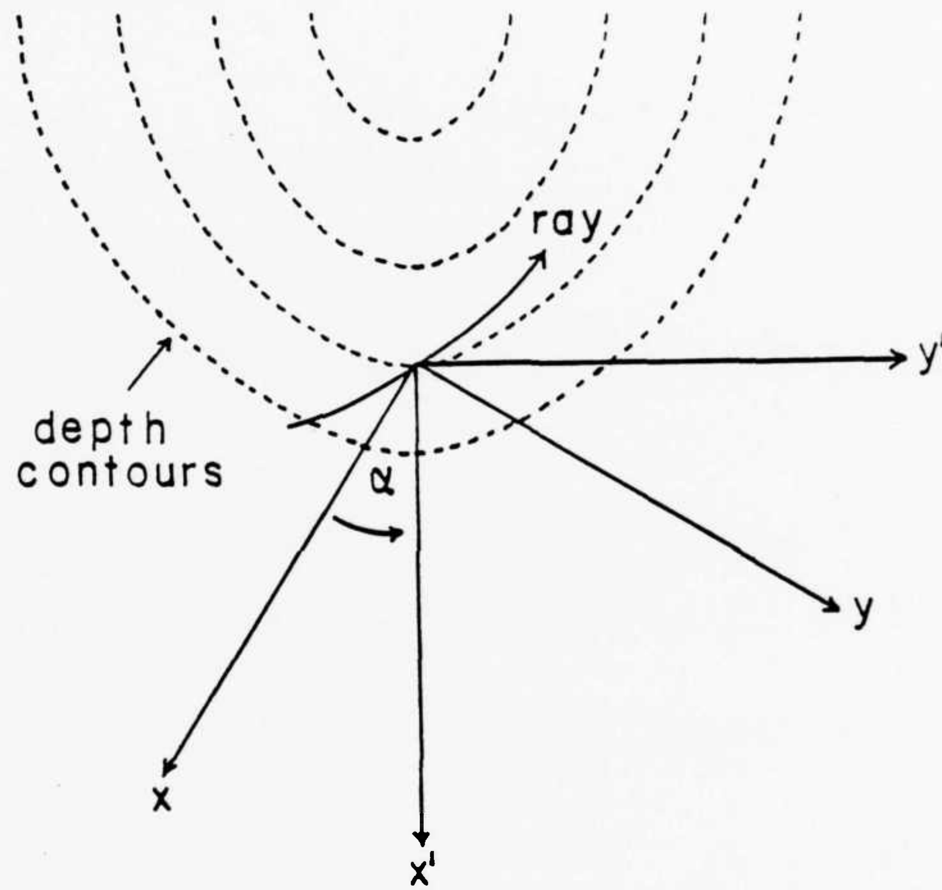


Figure (2-3). RELATIONSHIPS BETWEEN THE COORDINATE SYSTEMS AND THE WATER DEPTH CONTOURS

The partial derivatives of h in the x'y'-coordinate system with respect to the partial derivatives of h in the xy-coordinate system are given by

$$\frac{\partial h}{\partial x'} = \frac{\partial h}{\partial x} \cos\alpha + \frac{\partial h}{\partial y} \sin\alpha \tag{2-113}$$

$$\frac{\partial h}{\partial y'} = -\frac{\partial h}{\partial x} \sin\alpha + \frac{\partial h}{\partial y} \cos\alpha = 0 \tag{2-114}$$

$$\frac{\partial^2 h}{(\partial x')^2} = \frac{\partial^2 h}{\partial x^2} \cos^2\alpha + 2\frac{\partial^2 h}{\partial x \partial y} \sin\alpha \cos\alpha + \frac{\partial^2 h}{\partial y^2} \sin^2\alpha \tag{2-115}$$

$$\frac{\partial^2 h}{(\partial y')^2} = \frac{\partial^2 h}{\partial x^2} \sin^2\alpha - 2\frac{\partial h^2}{\partial x \partial y} \sin\alpha \cos\alpha + \frac{\partial^2 h}{\partial y^2} \cos^2\alpha \tag{2-116}$$

$$\frac{\partial^2 h}{\partial x' \partial y'} = \left(\frac{\partial^2 h}{\partial y^2} - \frac{\partial^2 h}{\partial x^2}\right) \sin\alpha \cos\alpha + \frac{\partial^2 h}{\partial x \partial y}\left(\cos^2\alpha - \sin^2\alpha\right) \tag{2-117}$$

c. Derivatives of v

As a convenience in the computations, the spatial derivatives of v are expressed in terms of the spatial derivatives of h. For linear theory (Lamb, 1932) the phase speed of a gravity water wave can be defined

$$v = \frac{g}{\omega} \tanh \frac{\omega h}{v} \tag{2-118}$$

where g is the acceleration due to gravity. The first partial derivatives of v in the x'y'-coordinate system are given by (Wilson, 1966; Dobson, 1967; Breeding, 1972)

$$\frac{\partial v}{\partial x'} = W \frac{\partial h}{\partial x'} \tag{2-119}$$

$$\frac{\partial v}{\partial y'} = W \frac{\partial h}{\partial y'} = 0 \tag{2-120}$$

where

$$W = \frac{g v (1 - a^2 v^2)}{v^2 + g h (1 - a^2 v^2)} \tag{2-121}$$

$$a = \frac{\omega}{g} \tag{2-122}$$

The second partial derivatives of v are defined by

$$\frac{\partial^2 v}{(\partial x')^2} = W \left[\frac{\partial^2 h}{(\partial x')^2} + Y_h \left(\frac{\partial h}{\partial x'} \right)^2 \right] \tag{2-123}$$

$$\frac{\partial^2 v}{(\partial y')^2} = W \frac{\partial^2 h}{(\partial y')^2} \tag{2-124}$$

$$\frac{\partial^2 v}{\partial x' \partial y'} = W \frac{\partial^2 h}{\partial x' \partial y'} \tag{2-125}$$

where

$$Y_h = -\frac{2gv^2}{\left[v^2 + gh(1 - \alpha^2 v^2)\right]^2} \tag{2-126}$$

d. Derivatives of U

For linear wave theory the collinear group speed of a gravity water wave can be defined

$$U = \frac{1}{2}\left(1 + \frac{I}{\sinh I}\right) v \tag{2-127}$$

where

$$I = \frac{2\omega h}{v} \tag{2-128}$$

The first partial derivatives of U in the x'y'-coordinate system are given by

$$\frac{\partial U}{\partial x'} = \frac{U}{v}\frac{\partial v}{\partial x'} + \eta \Phi \Psi_x \tag{2-129}$$

$$\frac{\partial U}{\partial y'} = \frac{U}{v}\frac{\partial v}{\partial y'} + \eta \Phi \Psi_y = 0 \tag{2-130}$$

where

$$\eta = \frac{U}{\upsilon} - \frac{1}{2} \tag{2-131}$$

$$\Phi = \left[1 - \left(I^2 + 4\eta^2\right)^{\frac{1}{2}}\right] \tag{2-132}$$

$$\Psi_x = \frac{\upsilon}{h}\frac{\partial h}{\partial x'} - \frac{\partial \upsilon}{\partial x'} \tag{2-133}$$

$$\Psi_y = \frac{\upsilon}{h}\frac{\partial h}{\partial y'} - \frac{\partial \upsilon}{\partial y'} = 0 \tag{2-134}$$

The second partial derivatives of U are given by

$$\frac{\partial^2 U}{(\partial x')^2} = \frac{U}{\upsilon}\frac{\partial^2 \upsilon}{(\partial x')^2} + \frac{\partial \eta}{\partial x'}\left(\frac{\partial \upsilon}{\partial x'} + \Phi\Psi_x\right) + \eta\left(\Psi_x\frac{\partial \Phi}{\partial x'} + \Phi\frac{\partial \Psi_x}{\partial x'}\right) \tag{2-135}$$

$$\frac{\partial^3 U}{(\partial y')^2} = \frac{U}{\upsilon}\frac{\partial^2 \upsilon}{(\partial y')^2} + \eta\Phi\frac{\partial \Psi_x}{\partial y'} \tag{2-136}$$

$$\frac{\partial^2 U}{\partial x' \partial y'} = \frac{U}{v} \frac{\partial^2 v}{\partial x' \partial y'} + \eta \Phi \frac{\partial \psi_x}{\partial y'} \tag{2-137}$$

where

$$\frac{\partial \eta}{\partial x'} = \frac{1}{v} \left(\frac{\partial U}{\partial x'} - \frac{U}{v} \frac{\partial v}{\partial x'} \right) \tag{2-138}$$

$$\frac{\partial \Phi}{\partial x'} = - \frac{\dfrac{I^2 \psi_x}{v} + 4 \eta \dfrac{\partial \eta}{\partial x'}}{\left(I^2 + 4 \eta^2 \right)^{\frac{1}{2}}} \tag{2-139}$$

$$\frac{\partial \psi_x}{\partial x'} = \frac{1}{h} \left(v \frac{\partial^2 h}{(\partial x')^2} - \psi_x \frac{\partial h}{\partial x'} \right) - \frac{\partial^2 v}{(\partial x')^2} \tag{2-140}$$

$$\frac{\partial \psi_y}{\partial y'} = \frac{v}{h} \frac{\partial^2 h}{(\partial y')^2} - \frac{\partial^2 v}{(\partial y')^2} \tag{2-141}$$

$$\frac{\partial \psi_x}{\partial y'} = \frac{v}{h} \frac{\partial^2 h}{\partial x' \partial y'} - \frac{\partial^2 v}{\partial x' \partial y'} \tag{2-142}$$

e. Derivatives of G

The first partial derivatives of G were derived in Section (2.1),b as Equations (2-22) and (2-23). In the x'y'-coordinate system these derivatives become

$$\frac{\partial G}{\partial x'} = \frac{\frac{\partial U}{\partial x'}\cos\phi + \frac{U}{v}\sin\phi\tan\gamma'\frac{\partial v}{\partial x'}}{1+\tan\phi\tan\theta'} \tag{2-143}$$

$$\frac{\partial G}{\partial y'} = 0 \tag{2-144}$$

The second partial derivatives of G can be expressed by

$$\frac{\partial^2 G}{(\partial x')^2} = \frac{1}{1+\tan\phi\tan\theta'}\left[\frac{\partial^2 U}{(\partial x')^2}\cos\phi - \frac{\partial\phi}{\partial x'}\left(2\frac{\partial U}{\partial x'}\sin\phi + G\frac{\partial\phi}{\partial x'}\right)\right.$$
$$\left. - U\sin\phi\left\{\tan\theta'\left(\frac{\partial\theta'}{\partial x'}\right)^2 - \tan\gamma'\left[\frac{1}{v}\frac{\partial^2 v}{(\partial x')^2} + \left(\frac{\partial\gamma'}{\partial x'}\right)^2\right]\right\}\right] \tag{2-145}$$

$$\frac{\partial^2 G}{(\partial y')^2} = \frac{\frac{\partial^2 U}{(\partial y')^2}\cos\phi - \frac{U}{v}\sin\phi\cot\gamma'\frac{\partial^2 v}{(\partial y')^2}}{1-\tan\phi\cot\theta'} \tag{2-146}$$

$$\frac{\partial^2 G}{\partial x'\partial y'} = \frac{\frac{\partial^2 U}{\partial x'\partial y'}\cos\phi + \frac{U}{v}\sin\phi\tan\gamma'\frac{\partial^2 v}{\partial x'\partial y'}}{1+\tan\phi\tan\theta'} \tag{2-147}$$

where

$$\frac{\partial \phi}{\partial x'} = \frac{\partial \theta'}{\partial x'} - \frac{\partial \gamma'}{\partial x'} \tag{2-148}$$

$$\frac{\partial \theta'}{\partial x'} = \frac{\tan \theta'}{G} \frac{\partial G}{\partial x'} \tag{2-149}$$

$$\frac{\partial \gamma'}{\partial x'} = \frac{\tan \gamma'}{v} \frac{\partial v}{\partial x'} \tag{2-150}$$

2.7 Spatial Derivatives of G, U, v, ω, u, and ε for Currents. In this section relations are presented for connecting the partial derivatives of G, U, v, ω, and a current of magnitude u and direction ε. The water depth is assumed to be constant. The spatial derivatives are determined in a coordinate system where the y-derivatives are reduced to simplified expressions. This results in a corresponding reduction in the number of computations.

a. Determination of u, ε, and their partial derivatives

For each ray point the current components u_x and u_y are interpolated from quadratic surface equations which are fitted to their respective current component values at 12 grid points as described for water depths in Section (2.6), a. The resulting quadratic equations are

$$u_x = E_{x1} + E_{x2} x + E_{x3} y + E_{x4} x^2 + E_{x5} xy + E_{x6} y^2 \tag{2-151}$$

$$u_y = E_{y1} + E_{y2} x + E_{y3} y + E_{y4} x^2 + E_{y5} xy + E_{y6} y^2 \tag{2-152}$$

The magnitude of the current is given by

$$u = (u_x^2 + u_y^2)^{\frac{1}{2}} \tag{2-153}$$

and the current direction is defined by

$$\tan \epsilon = \frac{u_y}{u_x} \tag{2-154}$$

The partial derivatives of u_x and u_y are found from Equations (2-151) and (2-152).

$$\frac{\partial u_x}{\partial x} = E_{x2} + 2E_{x4}x + E_{x5}y \tag{2-155}$$

$$\frac{\partial u_x}{\partial y} = E_{x3} + E_{x5}x + 2E_{x6}y \tag{2-156}$$

$$\frac{\partial^2 u_x}{\partial x^2} = 2E_{x4} \tag{2-157}$$

$$\frac{\partial^2 u_x}{\partial x \partial y} = E_{x5} \tag{2-158}$$

$$\frac{\partial^2 u_x}{\partial y^2} = 2E_{x6} \tag{2-159}$$

$$\frac{\partial u_y}{\partial x} = E_{y2} + 2E_{y4}x + E_{y5}y \tag{2-160}$$

$$\frac{\partial u_y}{\partial y} = E_{y3} + E_{y5}X + 2E_{y6}y \tag{2-161}$$

$$\frac{\partial^2 u_y}{\partial X^2} = 2E_{y4} \tag{2-162}$$

$$\frac{\partial^2 u_y}{\partial X \partial y} = E_{y5} \tag{2-163}$$

$$\frac{\partial^2 u_y}{\partial y^2} = 2E_{y6} \tag{2-164}$$

The partial derivatives of u are found to be given by

$$\frac{\partial u}{\partial X} = \frac{1}{u}\left(u_x \frac{\partial u_x}{\partial X} + u_y \frac{\partial u_y}{\partial X}\right) \tag{2-165}$$

$$\frac{\partial u}{\partial y} = \frac{1}{u}\left(u_x \frac{\partial u_x}{\partial y} + u_y \frac{\partial u_y}{\partial y}\right) \tag{2-166}$$

$$\frac{\partial^2 u}{\partial X^2} = \frac{1}{u}\left[u_x \frac{\partial^2 u_x}{\partial X^2} + \left(\frac{\partial u_x}{\partial X}\right)^2 + u_y \frac{\partial^2 u_y}{\partial X^2} + \left(\frac{\partial u_y}{\partial X}\right)^2 - \left(\frac{\partial u}{\partial X}\right)^2\right] \tag{2-167}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{1}{u}\left[u_x \frac{\partial^2 u_x}{\partial x \partial y} + \frac{\partial u_x}{\partial x}\frac{\partial u_x}{\partial y} + u_y \frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial u_y}{\partial x}\frac{\partial u_y}{\partial y} - \frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right] \tag{2-168}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{u}\left[u_x \frac{\partial^2 u_x}{\partial y^2} + \left(\frac{\partial u_x}{\partial y}\right)^2 + u_y \frac{\partial^2 u_y}{\partial y^2} + \left(\frac{\partial u_y}{\partial y}\right)^2 - \left(\frac{\partial u}{\partial y}\right)^2\right] \tag{2-169}$$

The partial derivatives of ε are determined to be

$$\frac{\partial \epsilon}{\partial x} = \frac{1}{u^2}\left(u_x \frac{\partial u_y}{\partial x} - u_y \frac{\partial u_x}{\partial x}\right) \tag{2-170}$$

$$\frac{\partial \epsilon}{\partial y} = \frac{1}{u^2}\left(u_x \frac{\partial u_y}{\partial y} - u_y \frac{\partial u_x}{\partial y}\right) \tag{2-171}$$

$$\frac{\partial^2 \epsilon}{\partial x^2} = \frac{1}{u}\left[\frac{1}{u}\left(u_x \frac{\partial^2 u_y}{\partial x^2} - u_y \frac{\partial^2 u_x}{\partial x^2}\right) - 2\frac{\partial \epsilon}{\partial x}\frac{\partial u}{\partial x}\right] \tag{2-172}$$

$$\frac{\partial^2 \epsilon}{\partial x \partial y} = \frac{1}{u}\left[\frac{1}{u}\left(\frac{\partial u_x}{\partial x}\frac{\partial u_y}{\partial y} + u_x \frac{\partial^2 u_y}{\partial x \partial y} - \frac{\partial u_y}{\partial x}\frac{\partial u_x}{\partial y} - u_y \frac{\partial^2 u_x}{\partial x \partial y}\right) - 2\frac{\partial \epsilon}{\partial y}\frac{\partial u}{\partial x}\right] \tag{2-173}$$

$$\frac{\partial^2 \epsilon}{\partial y^2} = \frac{1}{u}\left[\frac{1}{u}\left(u_x \frac{\partial^2 u_y}{\partial y^2} - u_y \frac{\partial^2 u_x}{\partial y^2}\right) - 2\frac{\partial \epsilon}{\partial y}\frac{\partial u}{\partial y}\right] \quad (2\text{-}174)$$

b. Rotation of axes to make computations

Following the procedure described in Section (2.6),b for water depths, the calculations are made in a x"y"-coordinate system where the x"-axis is taken in the direction of the gradient of the current speed. The ray particulars are tabulated in the xy-coordinate system which is fixed with respect to the current grids. The equations which relate these coordinate systems are expressed by

$$x'' = x \cos \alpha_c + y \sin \alpha_c \quad (2\text{-}175)$$

$$y'' = -x \sin \alpha_c + y \cos \alpha_c \quad (2\text{-}176)$$

$$\tan \alpha_c = \frac{\partial u}{\partial y} \Big/ \frac{\partial u}{\partial x} \quad (2\text{-}177)$$

where α_c is the angle by which the x"-axis is rotated with respect to the x-axis. Further

$$\rho'' = \rho - \alpha_c \quad (2\text{-}178)$$

$$\theta'' = \theta - \alpha_c \quad (2\text{-}179)$$

$$\gamma'' = \gamma - \alpha_c \tag{2-180}$$

The partial derivatives of u in the x"y"-coordinate system with respect to the partial derivatives of u in the xy-coordinate system are given by

$$\frac{\partial u}{\partial x''} = \frac{\partial u}{\partial x}\cos\alpha_c + \frac{\partial u}{\partial y}\sin\alpha_c \tag{2-181}$$

$$\frac{\partial u}{\partial y''} = -\frac{\partial u}{\partial x}\sin\alpha_c + \frac{\partial u}{\partial y}\cos\alpha_c = 0 \tag{2-182}$$

$$\frac{\partial^2 u}{(\partial x'')^2} = \frac{\partial^2 u}{\partial x^2}\cos^2\alpha_c + 2\frac{\partial^2 u}{\partial x\partial y}\sin\alpha_c\cos\alpha_c + \frac{\partial^2 u}{\partial y^2}\sin^2\alpha_c \tag{2-183}$$

$$\frac{\partial^2 u}{(\partial y'')^2} = \frac{\partial^2 u}{\partial x^2}\sin^2\alpha_c - 2\frac{\partial^2 u}{\partial x\partial y}\sin\alpha_c\cos\alpha_c + \frac{\partial^2 u}{\partial y^2}\cos^2\alpha_c \tag{2-184}$$

$$\frac{\partial^2 u}{\partial x''\partial y''} = \left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial x^2}\right)\sin\alpha_c\cos\alpha_c + \frac{\partial^2 u}{\partial x\partial y}(\cos^2\alpha_c - \sin^2\alpha_c) \tag{2-185}$$

The relations connecting the partial derivatives of ε between the two coordinate systems are given by

$$\frac{\partial \epsilon''}{\partial x''} = \frac{\partial \epsilon}{\partial x}\cos\alpha_c + \frac{\partial \epsilon}{\partial y}\sin\alpha_c \tag{2-186}$$

$$\frac{\partial \epsilon''}{\partial y''} = -\frac{\partial \epsilon}{\partial x} \sin \alpha_c + \frac{\partial \epsilon}{\partial y} \cos \alpha_c \tag{2-187}$$

$$\frac{\partial^2 \epsilon''}{(\partial x'')^2} = \frac{\partial^2 \epsilon}{\partial x^2} \cos^2 \alpha_c + 2\frac{\partial^2 \epsilon}{\partial x \partial y} \sin \alpha_c \cos \alpha_c + \frac{\partial^2 \epsilon}{\partial y^2} \sin^2 \alpha_c \tag{2-188}$$

$$\frac{\partial^2 \epsilon''}{(\partial y'')^2} = \frac{\partial^2 \epsilon}{\partial x^2} \sin^2 \alpha_c - 2\frac{\partial^2 \epsilon}{\partial x \partial y} \sin \alpha_c \cos \alpha_c + \frac{\partial^2 \epsilon}{\partial y^2} \cos^2 \alpha_c \tag{2-189}$$

$$\frac{\partial^2 \epsilon''}{\partial x'' \partial y''} = \left(\frac{\partial^2 \epsilon}{\partial y^2} - \frac{\partial^2 \epsilon}{\partial x^2}\right) \sin \alpha_c \cos \alpha_c + \frac{\partial^2 \epsilon}{\partial x \partial y}\left(\cos^2 \alpha_c - \sin^2 \alpha_c\right) \tag{2-190}$$

c. Derivatives of ω, v, and u_k

In Section (2.6), c the derivatives of v are expressed directly in terms of the corresponding derivatives of water depth. In considering currents the derivatives of v are related to derivatives in ω. For the first x"-derivatives it is found that

$$\frac{\partial \omega}{\partial x''} = -\left\{1 - \frac{k M_k W_\omega}{1+M_k} + \frac{u_k}{v}(1 - k W_\omega)\right\}^{-1}\left(\frac{k}{1+M_k}\right)\left[\frac{\partial u}{\partial x''} \cos(\gamma'' - \epsilon'') + u \sin(\gamma'' - \epsilon'')\frac{\partial \epsilon''}{\partial x''}\right] \tag{2-191}$$

$$\frac{\partial u_k}{\partial x''} = -\frac{M_k}{1+M_k}\frac{\partial v}{\partial x''} + \frac{1}{1+M_k}\left[\frac{\partial u}{\partial x''} \cos(\gamma'' - \epsilon'') + u \sin(\gamma'' - \epsilon'')\frac{\partial \epsilon''}{\partial x''}\right] \tag{2-192}$$

$$\frac{\partial v}{\partial x''} = W_\omega \frac{\partial \omega}{\partial x''} \tag{2-193}$$

where

$$W_\omega = \frac{hgv(1-a^2v^2) - v^3}{\omega[v^2 + gh(1-a^2v^2)]} \tag{2-194}$$

$$a = \frac{\omega}{g} \tag{2-195}$$

$$M_k = \frac{u \sin(\gamma'' - \epsilon'') \tan \gamma''}{v + u_k} \tag{2-196}$$

$$u_k = u \cos(\gamma'' - \epsilon'') \tag{2-197}$$

The first y"-derivatives can be expressed

$$\frac{\partial \omega}{\partial y''} = -\left\{1 + \frac{k M_\ell W_\omega}{1 - M_\ell} + \frac{u_k}{v}(1 - k W_\omega)\right\}^{-1} \left(\frac{k}{1 - M_\ell}\right) u \sin(\gamma'' - \epsilon'') \frac{\partial \epsilon''}{\partial y''} \tag{2-198}$$

$$\frac{\partial u_k}{\partial y''} = \frac{M_\ell}{1 - M_\ell} \frac{\partial v}{\partial y''} + \frac{1}{1 - M_\ell} u \sin(\gamma'' - \epsilon'') \frac{\partial \epsilon''}{\partial y''} \tag{2-199}$$

$$\frac{\partial v}{\partial y''} = W_\omega \frac{\partial \omega}{\partial y''} \tag{2-200}$$

where

$$M_\ell = \frac{u \sin(\gamma'' - \epsilon'') \cot \gamma''}{v + u_k} \tag{2-201}$$

The second x"-derivatives are determined to be

$$\frac{\partial^2 \omega}{(\partial x'')^2} = \left(1 - \frac{k M_k W_\omega}{1 + M_k} + \frac{u_k (1 - k W_\omega)}{v}\right)^{-1} \left[\frac{k M_k Y_\omega}{1 + M_k}\left(\frac{\partial \omega}{\partial x''}\right)^2 - k u_2 \right.$$
$$\left. - 2 \frac{\partial k}{\partial x''} \frac{\partial u_k}{\partial x''} + \frac{u_k}{v} \frac{\partial \omega}{\partial x''}\left(\frac{1 - k W_\omega}{v} \frac{\partial v}{\partial x''} + k Y_\omega \frac{\partial \omega}{\partial x''} + W_\omega \frac{\partial k}{\partial x''}\right)\right] \tag{2-202}$$

$$\frac{\partial^2 u_k}{(\partial x'')^2} = - \frac{M_k}{1 + M_k} \frac{\partial^2 v}{(\partial x'')^2} + u_2 \tag{2-203}$$

$$\frac{\partial^2 v}{(\partial x'')^2} = W_\omega \frac{\partial^2 \omega}{(\partial x'')^2} + Y_\omega \left(\frac{\partial \omega}{\partial x''}\right)^2 \tag{2-204}$$

where

$$Y_\omega = \frac{2v}{v^2 + gh(1 - a^2 v^2)}\left[\left(\frac{v}{\omega}\right)^2 + h(1 - a^2 v^2)\left\{(q - h\omega^2)\left(\frac{W_\omega}{v}\right)^2 \right.\right.$$
$$\left.\left. + 2h\omega\left(\frac{W_\omega}{v}\right) - \frac{1}{a\omega} - h\right\}\right] \tag{2-205}$$

$$u_2 = \frac{1}{1+M_k}\left[\frac{\partial^2 u}{(\partial X'')^2}\cos(\gamma''-\epsilon'') - 2\frac{\partial u}{\partial X''}\sin(\gamma''-\epsilon'')\left(\frac{\partial\gamma''}{\partial X''}-\frac{\partial\epsilon''}{\partial X''}\right)\right.$$

$$- u\cos(\gamma''-\epsilon'')\left(\frac{\partial\gamma''}{\partial X''}-\frac{\partial\epsilon''}{\partial X''}\right)^2 + u\sin(\gamma''-\epsilon'')\left\{-\tan\gamma''\left(\frac{\partial\gamma''}{\partial X''}\right)^2\right. \tag{2-206}$$

$$\left.\left.+\frac{\partial^2\epsilon''}{(\partial X'')^2}\right\}\right]$$

$$\frac{\partial\gamma''}{\partial X''} = \frac{\tan\gamma''}{v+u_k}\left(\frac{\partial v}{\partial X''}+\frac{\partial u_k}{\partial X''}\right) \tag{2-207}$$

$$\frac{\partial k}{\partial X''} = \frac{1-kW_\omega}{v}\,\frac{\partial\omega}{\partial X''} \tag{2-208}$$

The second derivatives with respect to y'' become

$$\frac{\partial^2\omega}{(\partial y'')^2} = \left(1+\frac{kM_l W_\omega}{1-M_l}+\frac{u_k(1-kW_\omega)}{v}\right)^{-1}\left[-\frac{kM_l Y_\omega}{1-M_l}\left(\frac{\partial\omega}{\partial y''}\right)^2 - k u_3\right.$$

$$\left.-2\frac{\partial k}{\partial y''}\frac{\partial u_k}{\partial y''}+\frac{u_k}{v}\frac{\partial\omega}{\partial y''}\left(\frac{1-kW_\omega}{v}\frac{\partial v}{\partial y''}+kY_\omega\frac{\partial\omega}{\partial y''}+W_\omega\frac{\partial k}{\partial y''}\right)\right] \tag{2-209}$$

$$\frac{\partial^2 u_k}{(\partial y'')^2} = \frac{M_l}{1-M_l}\,\frac{\partial^2 v}{(\partial y'')^2} + u_3 \tag{2-210}$$

$$\frac{\partial^2 v}{(\partial y'')^2} = W_\omega\frac{\partial^2\omega}{(\partial y'')^2} + Y_\omega\left(\frac{\partial\omega}{\partial y''}\right)^2 \tag{2-211}$$

where

$$u_3 = \frac{1}{1-M_\ell}\left[\frac{\partial^2 u}{(\partial y'')^2}\cos(\gamma''-\epsilon'') - u\cos(\gamma''-\epsilon'')\left(\frac{\partial\gamma''}{\partial y''} - \frac{\partial\epsilon''}{\partial y''}\right)^2 \right.$$

$$\left. + u\sin(\gamma''-\epsilon'')\left\{\cot\gamma''\left(\frac{\partial\gamma''}{\partial y''}\right)^2 + \frac{\partial^2\epsilon''}{(\partial y'')^2}\right\}\right] \qquad (2\text{-}212)$$

$$\frac{\partial\gamma''}{\partial y''} = -\frac{\cot\gamma''}{v+u_k}\left(\frac{\partial v}{\partial y''} + \frac{\partial u_k}{\partial y''}\right) \qquad (2\text{-}213)$$

$$\frac{\partial k}{\partial y''} = \frac{1-kW_\omega}{v}\frac{\partial\omega}{\partial y''} \qquad (2\text{-}214)$$

The mixed derivatives can be stated as

$$\frac{\partial^2\omega}{\partial x''\partial y''} = \left(1 - \frac{kM_kW_\omega}{1+M_k} + \frac{u_k(1-kW_\omega)}{v}\right)^{-1}\left[\frac{kM_kY_\omega}{1+M_k}\frac{\partial\omega}{\partial x''}\frac{\partial\omega}{\partial y''}\right.$$

$$- k u_4 - \frac{\partial k}{\partial y''}\frac{\partial u_k}{\partial x''} - \frac{\partial u_k}{\partial y''}\frac{\partial k}{\partial x''} + \frac{u_k}{v}\frac{\partial\omega}{\partial x''}\left(\frac{1-kW_\omega}{v}\frac{\partial v}{\partial y''}\right. \qquad (2\text{-}215)$$

$$\left.\left. + kY_\omega\frac{\partial\omega}{\partial y''} + W_\omega\frac{\partial k}{\partial y''}\right)\right]$$

$$\frac{\partial^2 u_k}{\partial x''\partial y''} = -\frac{M_k}{1+M_k}\frac{\partial^2 v}{\partial x''\partial y''} + u_4 \qquad (2\text{-}216)$$

$$\frac{\partial^2 v}{\partial x''\partial y''} = W_\omega\frac{\partial^2\omega}{\partial x''\partial y''} + Y_\omega\frac{\partial\omega}{\partial x''}\frac{\partial\omega}{\partial y''} \qquad (2\text{-}217)$$

where

$$u_4 = \frac{1}{1+M_g}\left[\frac{\partial^2 u}{\partial x''\partial y''}\cos(\gamma''-\epsilon'') - \frac{\partial u}{\partial x''}\sin(\gamma''-\epsilon'')\left(\frac{\partial \gamma''}{\partial y''} - \frac{\partial \epsilon''}{\partial y''}\right)\right.$$

$$- u\cos(\gamma''-\epsilon'')\left(\frac{\partial \gamma''}{\partial x''} - \frac{\partial \epsilon''}{\partial x''}\right)\left(\frac{\partial \gamma''}{\partial y''} - \frac{\partial \epsilon''}{\partial y''}\right) \tag{2-218}$$

$$\left. + u\sin(\gamma''-\epsilon'')\left\{-(\cot\gamma'' + 2\tan\gamma'')\frac{\partial \gamma''}{\partial x''}\frac{\partial \gamma''}{\partial y''} + \frac{\partial^2 \epsilon''}{\partial x''\partial y''}\right\}\right]$$

d. Derivatives of U

The spatial derivatives of U due to variations in a current are similar to the derivatives presented in Section (2.6), d for variations in U due to changes in water depth. The first derivatives are seen to be

$$\frac{\partial U}{\partial x''} = \frac{U}{v}\frac{\partial v}{\partial x''} + \eta\Phi\Delta_x \tag{2-219}$$

$$\frac{\partial U}{\partial y''} = \frac{U}{v}\frac{\partial v}{\partial y''} + \eta\Phi\Delta_y \tag{2-220}$$

where

$$\eta = \frac{U}{v} - \frac{1}{2} \tag{2-221}$$

$$\Phi = \left[1 - (I^2 + 4\eta^2)^{\frac{1}{2}}\right] \tag{2-222}$$

$$I = \frac{2\omega h}{v} \tag{2-223}$$

$$\Delta_x = \frac{v}{\omega}\frac{\partial \omega}{\partial x''} - \frac{\partial v}{\partial x''} \tag{2-224}$$

$$\Delta_y = \frac{v}{\omega}\frac{\partial \omega}{\partial y''} - \frac{\partial v}{\partial y''} \tag{2-225}$$

Note that η, Φ, and I are defined the same as in Section (2.6), d.

The second spatial derivatives of U are given by

$$\frac{\partial^2 U}{(\partial x'')^2} = \frac{U}{v}\frac{\partial^2 v}{(\partial x'')^2} + \frac{\partial \eta}{\partial x''}\left(\frac{\partial v}{\partial x''} + \Phi \Delta_x\right) + \eta\left(\Delta_x \frac{\partial \Phi}{\partial x''} + \Phi \frac{\partial \Delta_x}{\partial x''}\right) \tag{2-226}$$

$$\frac{\partial^2 U}{(\partial y'')^2} = \frac{U}{v}\frac{\partial^2 v}{(\partial y'')^2} + \frac{\partial \eta}{\partial y''}\left(\frac{\partial v}{\partial y''} + \Phi \Delta_y\right) + \eta\left(\Delta_y \frac{\partial \Phi}{\partial y''} + \Phi \frac{\partial \Delta_y}{\partial y''}\right) \tag{2-227}$$

$$\frac{\partial^2 U}{\partial x'' \partial y''} = \frac{U}{v}\frac{\partial^2 v}{\partial x'' \partial y''} + \frac{\partial \eta}{\partial y''}\left(\frac{\partial v}{\partial x''} + \Phi \Delta_x\right) + \eta\left(\Delta_x \frac{\partial \Phi}{\partial y''} + \Phi \frac{\partial \Delta_x}{\partial y''}\right) \tag{2-228}$$

where

$$\frac{\partial \eta}{\partial x''} = \frac{1}{v}\left(\frac{\partial U}{\partial x''} - \frac{U}{v}\frac{\partial v}{\partial x''}\right) \tag{2-229}$$

$$\frac{\partial \Phi}{\partial x''} = -\frac{\dfrac{I^2 \Delta_x}{v} + 4\eta \dfrac{\partial \eta}{\partial x''}}{\left(I^2 + 4\eta^2\right)^{\frac{1}{2}}} \tag{2-230}$$

$$\frac{\partial \Delta_x}{\partial x''} = \frac{1}{\omega}\left(v \frac{\partial^2 \omega}{(\partial x'')^2} - \Delta_x \frac{\partial \omega}{\partial x''}\right) - \frac{\partial^2 v}{(\partial x'')^2} \tag{2-231}$$

$$\frac{\partial \eta}{\partial y''} = \frac{1}{v}\left(\frac{\partial U}{\partial y''} - \frac{U}{v}\frac{\partial v}{\partial y''}\right) \tag{2-232}$$

$$\frac{\partial \Phi}{\partial y''} = -\frac{\dfrac{I^2 \Delta_y}{v} + 4\eta \dfrac{\partial \eta}{\partial y''}}{\left(I^2 + 4\eta^2\right)^{\frac{1}{2}}} \tag{2-233}$$

$$\frac{\partial \Delta_y}{\partial y''} = \frac{1}{\omega}\left(v \frac{\partial^2 \omega}{(\partial y'')^2} - \Delta_y \frac{\partial \omega}{\partial y''}\right) - \frac{\partial^2 v}{(\partial y'')^2} \tag{2-234}$$

$$\frac{\partial \Delta_x}{\partial y''} = \frac{1}{\omega}\left(v \frac{\partial^2 \omega}{\partial x'' \partial y''} - \Delta_y \frac{\partial \omega}{\partial x''}\right) - \frac{\partial^2 v}{\partial x'' \partial y''} \tag{2-235}$$

e. Derivatives of G and u_m

The first partial derivatives of G are considered in Section (2.1), b. It is necessary to consider the derivatives of G and u_m together. For the first x″-derivatives it is found that

$$\frac{\partial u_m}{\partial x''} = -\frac{M}{1+M}\frac{\partial G}{\partial x''} + \frac{1}{1+M}\left[\frac{\partial u}{\partial x''}\cos(\theta'' - \epsilon'') + u \sin(\theta'' - \epsilon'')\frac{\partial \epsilon''}{\partial x''}\right] \tag{2-236}$$

$$\frac{\partial G}{\partial x''} = \left(1 + \frac{N_\theta}{1+M}\right)^{-1}\left[\frac{\partial U}{\partial x''}\cos\phi - \frac{N_\theta}{1+M}\left\{\frac{\partial u}{\partial x''}\cos(\theta'' - \epsilon'') + u \sin(\theta'' - \epsilon'')\frac{\partial \epsilon''}{\partial x''}\right\} + N_\gamma\left(\frac{\partial v}{\partial x''} + \frac{\partial u_m}{\partial x''}\right)\right] \tag{2-237}$$

where

$$M = \frac{u \sin(\theta'' - \epsilon'') \tan\theta''}{G + u_m} \tag{2-238}$$

$$N_\theta = \frac{U \sin\phi \tan\theta''}{G + u_m} \tag{2-239}$$

$$N_\gamma = \frac{U \sin\phi \tan\gamma''}{v + u_k} \tag{2-240}$$

$$u_m = u \cos(\theta'' - \epsilon'') \tag{2-241}$$

The y"-derivatives can be stated as

$$\frac{\partial u_m}{\partial y''} = \frac{M_y}{1 - M_y}\frac{\partial G}{\partial y''} + \frac{1}{1 - M_y}\left[u \sin(\theta'' - \epsilon'')\frac{\partial \epsilon''}{\partial y''}\right] \tag{2-242}$$

$$\frac{\partial G}{\partial y''} = \left(1 - \frac{J_\theta}{1 - M_y}\right)^{-1}\left[\frac{\partial U}{\partial y''}\cos\phi + \frac{J_\theta}{1 - M_y}\left\{u \sin(\theta'' - \epsilon'')\frac{\partial \epsilon''}{\partial y''}\right\} - J_\gamma\left(\frac{\partial v}{\partial y''} + \frac{\partial u_k}{\partial y''}\right)\right] \tag{2-243}$$

where

$$M_{\gamma} = \frac{u \sin(\theta'' - \epsilon'') \cot\theta''}{G + u_m} \tag{2-244}$$

$$J_{\theta} = \frac{U \sin\phi \cot\theta''}{G + u_m} \tag{2-245}$$

$$J_{\gamma} = \frac{U \sin\phi \cot\gamma''}{v + u_k} \tag{2-246}$$

The second x''-derivatives are determined to be

$$\frac{\partial^2 u_m}{(\partial x'')^2} = -\frac{M}{1+M}\frac{\partial^2 G}{(\partial x'')^2} + G_2 \tag{2-247}$$

$$\begin{aligned}\frac{\partial^2 G}{(\partial x'')^2} = \left(1 + \frac{N_\theta}{1+M}\right)^{-1} \Bigg[& \frac{\partial^2 U}{(\partial x'')^2}\cos\phi - \frac{\partial\phi}{\partial x''}\left(2\frac{\partial U}{\partial x''}\sin\phi \right. \\ & \left. + G\frac{\partial\phi}{\partial x''}\right) - N_\theta G_2 - U\sin\phi\tan\theta''\left(\frac{\partial\theta''}{\partial x''}\right)^2 \\ & + N_\gamma\left(\frac{\partial^2 v}{(\partial x'')^2} + \frac{\partial^2 u_k}{(\partial x'')^2}\right) + U\sin\phi\tan\gamma''\left(\frac{\partial\gamma''}{\partial x''}\right)^2\Bigg]\end{aligned} \tag{2-248}$$

where

$$G_2 = \frac{1}{1+M}\left[\frac{\partial^2 u}{(\partial x'')^2}\cos(\theta''-\epsilon'') - 2\frac{\partial u}{\partial x''}\sin(\theta''-\epsilon'')\left(\frac{\partial\theta''}{\partial x''}-\frac{\partial\epsilon''}{\partial x''}\right)\right.$$

$$- u\cos(\theta''-\epsilon'')\left(\frac{\partial\theta''}{\partial x''}-\frac{\partial\epsilon''}{\partial x''}\right)^2 + u\sin(\theta''-\epsilon'')\left\{-\tan\theta''\left(\frac{\partial\theta''}{\partial x''}\right)^2\right. \quad (2\text{-}249)$$

$$\left.\left.+\frac{\partial^2\epsilon''}{(\partial x'')^2}\right\}\right]$$

$$\frac{\partial\theta''}{\partial x''} = \frac{\tan\theta''}{G+u_m}\left(\frac{\partial G}{\partial x''}+\frac{\partial u_m}{\partial x''}\right) \quad (2\text{-}250)$$

$$\frac{\partial\phi}{\partial x''} = \frac{\partial\theta''}{\partial x''} - \frac{\partial\gamma''}{\partial x''} \quad (2\text{-}251)$$

The second derivatives with respect to y" are

$$\frac{\partial^2 u_m}{(\partial y'')^2} = \frac{M_y}{1-M_y}\frac{\partial^2 G}{(\partial y'')^2} + G_3 \quad (2\text{-}252)$$

$$\frac{\partial^2 G}{(\partial y'')^2} = \left(1-\frac{J_\theta}{1-M_y}\right)^{-1}\left[\frac{\partial^2 U}{(\partial y'')^2}\cos\phi - \frac{\partial\phi}{\partial y''}\left(2\frac{\partial U}{\partial y''}\sin\phi\right.\right.$$

$$\left. + G\frac{\partial\phi}{\partial y''}\right) + J_\theta G_3 + U\sin\phi\cot\theta''\left(\frac{\partial\theta''}{\partial y''}\right)^2 \quad (2\text{-}253)$$

$$\left. - J_\gamma\left(\frac{\partial^2 v}{(\partial y'')^2}+\frac{\partial^2 u_s}{(\partial y'')^2}\right) - U\sin\phi\cot\gamma''\left(\frac{\partial\gamma''}{\partial y''}\right)^2\right]$$

where

$$G_3 = \frac{1}{1-M_y}\left[\frac{\partial^2 u}{(\partial y'')^2}\cos(\theta''-\epsilon'') - u\cos(\theta''-\epsilon'')\left(\frac{\partial\theta''}{\partial y''}-\frac{\partial\epsilon''}{\partial y''}\right)^2 + u\sin(\theta''-\epsilon'')\left\{\cot\theta''\left(\frac{\partial\theta''}{\partial y''}\right)^2+\frac{\partial\epsilon''}{(\partial y'')^2}\right\}\right] \tag{2-254}$$

$$\frac{\partial\theta''}{\partial y''} = -\frac{\cot\theta''}{G+u_m}\left(\frac{\partial G}{\partial y''}+\frac{\partial u_m}{\partial y''}\right) \tag{2-255}$$

$$\frac{\partial\phi}{\partial y''} = \frac{\partial\theta''}{\partial y''} - \frac{\partial\gamma''}{\partial y''} \tag{2-256}$$

The mixed derivatives can be stated as

$$\frac{\partial^2 u_m}{\partial x''\partial y''} = -\frac{M_{xy}}{1+M_{xy}}\frac{\partial^2 G}{\partial x''\partial y''} + G_4 \tag{2-257}$$

$$\frac{\partial^2 G}{\partial x''\partial y''} = \left(1+\frac{N_\theta}{1+M_{xy}}\right)^{-1}\left[\frac{\partial^2 U}{\partial x''\partial y''}\cos\phi - \frac{\partial\phi}{\partial y''}\left(\frac{\partial U}{\partial x''}\sin\phi + G\frac{\partial\phi}{\partial x''}\right) - \frac{\partial U}{\partial y''}\sin\phi\frac{\partial\phi}{\partial x''} - N_\theta G_4 - U\sin\phi\frac{\partial\theta''}{\partial x''}\frac{\partial\theta''}{\partial y''}\left(\cot\theta'' + 2\tan\theta''\right) + N_\gamma\left(\frac{\partial^2 v}{\partial x''\partial y''}+\frac{\partial^2 u_s}{\partial x''\partial y''}\right) + U\sin\phi\frac{\partial\gamma''}{\partial x''}\frac{\partial\gamma''}{\partial y''}\left(\cot\gamma''+2\tan\gamma''\right)\right] \tag{2-258}$$

where

$$G_4 = \frac{1}{1+M_{xy}}\left[\frac{\partial^2 u}{\partial x''\partial y''}\cos(\theta''-\epsilon'') - \left(\frac{\partial\theta''}{\partial y''} - \frac{\partial\epsilon''}{\partial y''}\right)\left\{\frac{\partial u}{\partial x''}\sin(\theta''-\epsilon'') + u\cos(\theta''-\epsilon'')\left(\frac{\partial\theta''}{\partial x''} - \frac{\partial\epsilon''}{\partial x''}\right)\right\} + u\cos(\theta''-\epsilon'')\left\{-\frac{\partial\theta''}{\partial x''}\frac{\partial\theta''}{\partial y''}\left(\cot\theta'' + 2\tan\theta''\right) + \frac{\partial^2\epsilon''}{\partial x''\partial y''}\right\}\right] \tag{2-259}$$

$$M_{xy} = \frac{u\cos(\theta''-\epsilon'')\tan\theta''}{G+u_m} \tag{2-260}$$

2.8 Summary of Basic Equations. In the x'y'- and x"y"-coordinate systems, Equation (2-24) for the ray curvature of the wave packet becomes

$$K_G = \frac{1}{G+u_m}\left[\left\{\sin\theta'\frac{\partial G}{\partial x'}\right\}_h - \left\{\sin\theta''\left(\frac{\partial G}{\partial x''} + \frac{\partial u_m}{\partial x''}\right) - \cos\theta''\left(\frac{\partial G}{\partial y''} + \frac{\partial u_m}{\partial y''}\right)\right\}_\omega\right] \tag{2-261}$$

Equation (2-25), which defines the ratio of the incremental distances of the wave packets and rays, is applied separately to the calculations accounting for water depth and current variations.

Equation (2-27) for the change in wavelet direction becomes

$$\Delta\gamma = \left[\left\{\cos\rho'\tan\gamma'\frac{\partial v}{\partial x'}\right\}_h + \left\{\cos\rho''\left(\tan\gamma''\left(\frac{\partial v}{\partial x''} + \frac{\partial u_k}{\partial x''}\right) - \left(\frac{\partial v}{\partial y''} + \frac{\partial u_k}{\partial y''}\right)\right)\right\}_\omega\right]\frac{G_k\Delta t}{v+u_k} \tag{2-262}$$

When the wavelet direction is computed using Snell's law, the x'y'-coordinate system is the natural system to use for variations in water depth, and the x"y"-coordinate system is used for changes due to currents. Snell's law is stated in Equations (2-28) and (2-29). The Doppler shifted frequency, due to a current, is defined by Equation (2-13).

The wave height is given by Equation (2-54). The shoaling coefficient is defined by Equation (2-51), whereas the friction coefficient is defined by Equations (2-98), (2-99), and (2-56). The refraction coefficient is

determined as a function of β using Equation (2-58). The ray separation factor β is determined by solving Equation (2-59) where

$$p = -2\left[\left\{\cos\theta' \frac{\partial G}{\partial x'}\right\}_h + \left\{\cos\theta''\left(\frac{\partial G}{\partial x''} + \frac{\partial u_m}{\partial x''}\right) + \sin\theta''\left(\frac{\partial G}{\partial y''} + \frac{\partial u_m}{\partial y''}\right)\right\}_\omega\right] \tag{2-263}$$

$$\begin{aligned} q = G_T\Bigg[&\left\{\sin^2\theta' \frac{\partial^2 G}{(\partial x')^2} - 2\sin\theta'\cos\theta' \frac{\partial^2 G}{\partial x'\partial y'} + \cos^2\theta' \frac{\partial^2 G}{(\partial y')^2}\right\}_h \\ &+ \left\{\sin^2\theta''\left(\frac{\partial^2 G}{(\partial x'')^2} + \frac{\partial^2 u_m}{(\partial x'')^2}\right) - 2\sin\theta''\cos\theta''\left(\frac{\partial^2 G}{\partial x''\partial y''} + \frac{\partial^2 u_m}{\partial x''\partial y''}\right)\right. \\ &\left. + \cos^2\theta''\left(\frac{\partial^2 G}{(\partial y'')^2} + \frac{\partial^2 u_m}{(\partial y'')^2}\right)\right\}_\omega\Bigg] \end{aligned} \tag{2-264}$$

In the x'y'-coordinate system $\partial h/\partial y' = 0$. As a result the first partial derivatives of the wave speeds with respect to y' vanish, and there is a simplification in the second derivatives involving y'. In the x"y"-coordinate system $\partial u/\partial y'' = 0$. This results in a simplification of all the derivatives involving y". However, the first partial derivatives of the speeds do not vanish unless $\partial\varepsilon'/\partial y'' = 0$.

CHAPTER III THE COMPUTER PROGRAM

3.1 Introduction to the Computer Program. Including the arrays the computer program requires approximately 580,000 bytes of storage. Of this amount, the arrays CMAT, CURX, CURY, AX, and AY occupy about 187,000 bytes of storage.

The input and output directions of the wave packets, wavelets, and currents are defined as the directions from which the waves and currents come with respect to true north. Before making calculations these angles are transformed using the following relationships

$$\theta_C = \mathrm{CNVRSA} - \theta_N + 180 \qquad (3\text{-}1)$$

$$\gamma_C = \mathrm{CNVRSA} - \gamma_N + 180 \qquad (3\text{-}2)$$

$$\varepsilon_C = \mathrm{CNVRSA} - \varepsilon_N + 180 \qquad (3\text{-}3)$$

where the subscript C refers to the calculation coordinate system, the subscript N denotes the true north coordinate system, and CNVRSA is the direction of the positive x-axis of the grids with respect to true north. The angles are in degrees.

The program listing contains information on the input and output devices used in running the program. All the input and output parameters are defined. A description is given of the subroutine structure. Notes are provided on the plotting software which is used. The use of double precision in the calculations is discussed. Numerous comments are included to explain the operation of the program.

3.2 Listing of the Computer Program.

```
      PROGRAM WAVPAK
C
C THIS IS A PROGRAM FOR CALCULATING AND PLOTTING THE PATHS OF SURFACE
C GRAVITY WATER WAVE PACKETS AND FOR CALCULATING THE WAVE HEIGHTS ALONG
C THESE PATHS CONSIDERING WATER DEPTHS AND CURRENTS.  THE EFFECTS OF
C SHOALING, REFRACTION, AND ENERGY DISSIPATION ARE ACCOUNTED FOR.
C
C THE PROGRAM WAS COMPLETED SEPTEMBER 1982 UNDER A CONTRACT WITH THE
C COASTAL SCIENCES PROGRAM, OFFICE OF NAVAL RESEARCH.  THE PROGRAM WAS
C PREPARED BY
C
C       J. ERNEST BREEDING, JR
C       SHELLEY KAY HORTON
C          DEPARTMENT OF OCEANOGRAPHY AND OCEAN ENGINEERING
C
C       MICHAEL C. NEWELL
C          INTERACTIVE COMPUTING FACILITY
C
C          FLORIDA INSTITUTE OF TECHNOLOGY
C          MELBOURNE, FL      32901
C          USA
C
C THIS PROGRAM IS BASED ON A PROGRAM FOR COMPUTING THE PATHS OF MONOCHROMATIC
C RAYS DUE TO VARIATIONS IN WATER DEPTH BY
C
C       W. STANLEY WILSON, 'A METHOD FOR CALCULATING AND PLOTTING SURFACE
C       WAVE RAYS,' TECHNICAL MEMORANDUM NO. 17, COASTAL ENGINEERING
C       RESEARCH CENTER, 57 PP. (1966) (AD-636-771).
C
C WITH THE EXCEPTION OF THE PLOTTING SUBROUTINES, THE WILSON PROGRAM WAS
C EXTENSIVELY MODIFIED IN ORDER TO COMPUTE THE PATH OF A WAVE PACKET AND
C TO COMPUTE THE WAVE HEIGHT.
C
C COPIES OF THIS PROGRAM MAY BE OBTAINED ON 9-TRACK, 800 OR 1600 BPI ASCII
C ENCODED TAPES BY SENDING A BLANK TAPE TO THE AUTHORS.
C
C
C I/O UNIT NUMBERS
C
C       UNIT 1          CONTROL DATA FILE (INPUT)
C
C       UNIT 2          WATER DEPTH GRID FILE (INPUT)
C
C       UNIT 3          CURRENT SPEED GRID FILE (INPUT)
C
C       UNIT 5          TERMINAL INPUT (USED ONLY BY SUBROUTINE IOSET)
C
C       UNIT 6          PRINT FILE (OUTPUT)
```

```
C
C        UNIT 9           PLOT FILE (OUTPUT)
C
C  INPUT PARAMETERS
C
C        A                IS THE INITIAL DIRECTION FROM WHICH THE WAVE PACKET
C                         COMES WITH RESPECT TO TRUE NORTH.
C
C        AKRTOL           DETERMINES THE ACCURACY IN CALCULATING THE REFRACTION
C                         COEFFICIENT.
C
C        AV               IS THE INITIAL DIRECTION FROM WHICH THE WAVELETS COME
C                         WITH RESPECT TO TRUE NORTH.
C
C        CCON             IS A FACTOR TO CONVERT THE CURRENTS IN CURX AND CURY TO
C                         FEET/SECOND OR METERS/SECOND.
C
C        CF               IS THE FRICTION FACTOR FOR THE FRICTION COEFFICIENT.
C
C        CIN              IF CIN IS NOT ZERO IT IS THE TRAVEL TIME IN SECONDS
C                         BETWEEN SUCCESSIVE TICK MARKS ON A RAY.
C
C        CMAT             IS THE WATER DEPTH GRID.
C
C        CNVRSA           IS THE DIRECTION OF THE POSITIVE X-AXIS OF THE WATER
C                         DEPTH AND CURRENT GRIDS WITH RESPECT TO TRUE NORTH.
C
C        CONTURC          SPECIFIES THE SOUNDING CURRENTS IN FEET/SECOND OR
C                         METERS/SECOND.
C
C        CONTURD          SPECIFIES THE SOUNDING DEPTHS IN FEET OR METERS.
C
C        CURX             IS THE X-COMPONENT CURRENT GRID.
C
C        CURY             IS THE Y-COMPONENT CURRENT GRID.
C
C        DATE1,
C          DATE2          DEFINE THE YEAR, MONTH, AND DAY.
C
C        DCON             IS A FACTOR TO CONVERT THE WATER DEPTHS IN CMAT
C                         TO FEET OR METERS.
C
C        DELTAT           IS THE TIME STEP IN SECONDS.
C
C        DEP              IS THE WATER DEPTH IN FEET OR METERS IF THERE IS
C                         NO WATER DEPTH GRID.
C
C        DIR              IS A COMPUTER RUN IDENTIFIER.
C
C        GRID             IS THE NUMBER OF FEET OR METERS PER GRID UNIT.
C
C        HGTZ             IS THE INITIAL WAVE HEIGHT IN FEET OR METERS.
C
```

```
C         HT                IS THE HEIGHT OF THE PLOT IN INCHES OR CENTIMETERS.
C
C         MM                IS THE MAXIMUM X FOR THE WATER DEPTH AND CURRENT
C                           GRIDS.
C
C         MOE               IS THE UNITS SPECIFIER
C                              = 0 => ENGLISH UNITS
C                              = 1 => METRIC UNITS
C
C         MXPLOT            IS THE NUMBER OF PLOTS OR COMPUTER RUNS.
C
C         NAX               IS THE PLOT AXES CALIBRATE FLAG
C                              = 0 => DO NOT CALIBRATE AXES
C                              = 1 => CALIBRATE AXES
C
C         NC                IS THE CURRENT GRID FLAG
C                              = 0 => THERE IS NO CURRENT GRID
C                              = 1 => THERE IS A CURRENT GRID
C
C         NCC               IF NONZERO, IT IS THE NUMBER OF SOUNDING CURRENT
C                           VALUES FOR A PLOT.
C
C         NCO               IF NONZERO, IT IS THE NUMBER OF SOUNDING WATER DEPTH
C                           VALUES FOR A PLOT.
C
C         ND                IS THE WATER DEPTH GRID FLAG
C                              = 0 => THERE IS NO WATER DEPTH GRID
C                              = 1 => THERE IS A WATER DEPTH GRID
C
C         NFAN              IS THE RAY NUMBER FLAG
C                              = 0 => RAYS ARE NUMBERED AT THEIR INITIAL POINTS
C                              = 1 => RAYS ARE NUMBERED AT THEIR TERMINAL POINTS
C
C         NN                IS THE MAXIMUM Y FOR THE WATER DEPTH AND CURRENT
C                           GRIDS.
C
C         NNSKIP            IS THE AMOUNT ADDED TO THE Y-COLUMN IN SELECTING THE
C                           NEXT COLUMN FOR LOCATING SOUNDING VALUES.
C
C         NOR               IS THE NUMBER OF RAYS FOR A GIVEN RUN.
C
C         NPT               IS THE FLAG FOR OPTIONAL RAY OUTPUT
C                              = 0 => OPTIONAL RAY PARTICULARS ARE NOT PRINTED
C                              = 1 => OPTIONAL RAY PARTICULARS ARE PRINTED
C
C         NROPT             IS THE REFLECTION POINT CONTINUATION FLAG
C                              = 0 => RAY IS NOT CONTINUED BEYOND REFLECTION POINT
C                              = 1 => RAY IS CONTINUED BEYOND REFLECTION POINT
C
C         NSH               IS THE SHORELINE FLAG
C                              = 0 => SHORELINE IS NOT DRAWN
C                              = 1 => SHORELINE IS DRAWN
C
```

```
C        NSK               DETERMINES THE FREQUENCY OF PRINTED OUTPUT.  OUTPUT
C                          OCCURS FOR THOSE VALUES OF MAX WHICH ARE AN INTEGRAL
C                          MULTIPLE OF NSK.
C
C        NWBRK             IS THE WAVE BREAK TEST FLAG
C                             = 0 => WAVE BREAK TEST IS MADE
C                             = 1 => WAVE BREAK TEST IS NOT MADE
C
C        NXCMAT            IS THE GRID READ FLAG
C                             = 0 => READ NEW GRID(S)
C                             = 1 => USE GRID(S) FROM PREVIOUS PLOT
C
C        PROJCT            IS A COMPUTER RUN IDENTIFIER.
C
C        TT                IS THE INPUT WAVELET.PERIOD IN SECONDS.
C
C        X,Y               ARE THE INITIAL RAY COORDINATES.
C
C        Z                 IS THE CURRENT SPEED IN FEET/SECOND OR METERS/
C                          SECOND IF THERE ARE NO CURRENT GRIDS.
C
C        ZD                IS THE INITIAL DIRECTION FROM WHICH THE CURRENT COMES
C                          WITH RESPECT TO TRUE NORTH IF THERE ARE NO CURRENT
C                          GRIDS.
C
C OUTPUT RAY PARTICULARS
C
C        CUR:DI            IS THE DIRECTION FROM WHICH THE CURRENT COMES WITH
C                          RESPECT TO TRUE NORTH.
C
C        CUR:SP            IS THE CURRENT SPEED IN FEET/SECOND OR METERS/SECOND.
C
C        DEPTH             IS THE WATER DEPTH IN FEET OR METERS.
C
C        G=U *
C         COS(PACK-WAVE)   IS THE GEOMETRIC GROUP SPEED IN FEET/SECOND
C                          OR METERS/SECOND RELATIVE TO THE CURRENT.
C
C        GR                IS THE RAY SPEED IN FEET/SECOND OR METERS/SECOND.
C
C        HGT               IS THE WAVE HEIGHT IN FEET OR METERS.
C
C        KF                IS THE FRICTION COEFFICIENT.
C
C        KR                IS THE REFRACTION COEFFICIENT.
C
C        KS                IS THE SHOALING COEFFICIENT.
C
C        MAX               IS AN INDEX TO NUMBER POINTS ALONG A RAY.
C
C        PACK              IS THE DIRECTION FROM WHICH THE WAVE PACKET COMES WITH
C                          RESPECT TO TRUE NORTH.
C
```

```
C       PERIOD          IS THE WAVELET PERIOD IN SECONDS RELATIVE TO THE
C                       CURRENT.
C
C       RAY             IS THE DIRECTION FROM WHICH THE RAY COMES WITH RESPECT
C                       TO TRUE NORTH.
C
C       WAVE            IS THE DIRECTION FROM WHICH THE WAVELETS (IN A PACKET)
C                       COME WITH RESPECT TO TRUE NORTH.
C
C       X,Y             ARE THE COORDINATES OF A RAY POINT.
C
C ADDITIONAL OUTPUT IF NPT IS NOT ZERO
C
C       BETA            IS THE RAY SEPARATION FACTOR.
C
C       BRK UP          IS THE TYPE OF TIME STEP BREAKUP IF ONE OCCURS. IF THE
C                       BREAKUP OCCURS IN ORDER TO MAINTAIN ACCURACY IN THE
C                       REFRACTION CALCULATIONS, 'BETA' APPEARS IN THE
C                       PRINTOUT.  IF A TIME STEP BREAKUP IS REQUIRED TO KEEP
C                       THE CHANGE IN 'PACK' TO LESS THAN ONE DEGREE BETWEEN
C                       SUCCESSIVE RAY POINTS NEAR A REFLECTION POINT,
C                       'REFLECT' APPEARS IN THE PRINTOUT.
C
C       CURVATURE       IS THE RAY CURVATURE OF THE PACKET IN RADIANS/GRID
C                       UNIT.
C
C       DBETA/DT        IS THE TIME DERIVATIVE OF BETA.
C
C       GT              IS THE SPEED OF THE ADVECTED GROUP FRONT IN
C                       FEET/SECOND OR METERS/SECOND RELATIVE TO THE
C                       CURRENT.
C
C       NO              IS THE NUMBER OF INTERVALS THE INPUT TIME STEP IS
C                       DIVIDED INTO.
C
C       PCT:CX          IS THE MAXIMUM PERCENTAGE DIFFERENCE FOR THE X-
C                       COMPONENT CURRENT GRID. SEE EXPLANATION TO PCT:D.
C
C       PCT:CY          IS THE MAXIMUM PERCENTAGE DIFFERENCE FOR THE Y-
C                       COMPONENT CURRENT GRID. SEE EXPLANATION TO PCT:D.
C
C       PCT:D           IS THE MAXIMUM OF THE PERCENTAGE DIFFERENCES AT THE 4
C                       GRID POINTS CLOSEST TO THE RAY POINT OF THE SURFACE
C                       FIT DERIVED WATER DEPTH RELATIVE TO THE ACTUAL DEPTH.
C
C       ROTAT:C         IS THE ANGLE OF THE ROTATED XY-SYSTEM (WHERE THE Y-
C                       DERIVATIVES ARE SIMPLIFIED) RELATIVE TO THE CURRENT
C                       GRID XY-SYSTEM.
C
C       ROTAT:D         IS THE ANGLE OF THE ROTATED XY-SYSTEM (WHERE THE FIRST
C                       Y-DERIVATIVES VANISH) RELATIVE TO THE WATER DEPTH
C                       GRID XY-SYSTEM.
C
```

```
C       U                 IS THE CONVENTIONAL GROUP SPEED IN FEET/SECOND OR
C                         METERS/SECOND RELATIVE TO THE CURRENT.
C
C       V                 IS THE PHASE SPEED IN FEET/SECOND OR METERS/SECOND
C                         RELATIVE TO THE CURRENT.
C
C       VT                IS THE SPEED OF THE ADVECTED WAVELET FRONT IN FEET/
C                         SECOND OR METERS/SECOND RELATIVE TO THE CURRENT.
C
C
C
C  SUBROUTINE STRUCTURE
C
C       WAVPAK                      MAINLINE PROGRAM
C
C            IOSET                       SET UP I/O UNITS
C
C            NUMCON                      PLOT SOUNDING DEPTHS
C               NUMBER                      CALCOMP ROUTINE TO PLOT NUMBERS
C               PLOT                        CALCOMP ROUTINE TO PLOT SYMBOLS
C
C            NUMCON2                     PLOT CURRENT SPEED CONTURS
C               NUMBER                      CALCOMP ROUTINE TO PLOT NUMBERS
C               PLOT                        CALCOMP ROUTINE TO PLOT POINTS
C
C            PLOT                        CALCOMP ROUTINE TO PLOT POINTS
C
C            PLOTS                       CALCOMP ROUTINE TO INITIALIZE PLOTTER
C
C            PRTPRM                      PRINT OUT INPUT PARAMETERS
C
C            RAYN                        CONTROL RAY CALCULATIONS AND PRINTOUT
C               ANGCON                      CONVERT ANGLES FOR PRINTOUT
C               DRAW                        DRAW A RAY PATH AND TICK MARKS
C                  NUMBER                      CALCOMP ROUTINE TO DRAW NUMBERS
C                  PLOT                        CALCOMP ROUTINE TO PLOT POINTS
C               HEIGHT                      COMPUTE WAVE HEIGHT
C               MOVE                        MOVE PACKET ALONG PATH
C                  ANGCON                      CONVERT ANGLES FOR PRINTOUT
C                  HEIGHT                      COMPUTE WAVE HEIGHT
C                  PAGCOL                      PRINT PAGE AND COLUMN HEADINGS
C                  SURFCE                      COMPUTE WAVE PARTICULARS
C                     VELCTY                      COMPUTE WAVE SPEEDS
C               PAGCOL                      PRINT PAGE AND COLUMN HEADINGS
C               PCD                         COMPUTE PERCENT DIFFERENCES
C               STORE                       STORE DATA ON CURRENT POINT
C               SURFCE                      COMPUTE WAVE PARTICULARS
C                  VELCTY                      COMPUTE WAVE SPEEDS
C
C            SHORE                       PLOT IN THE SHORELINE IF REQUIRED
```

```
C                    PLOT                          CALCOMP ROUTINE TO PLOT POINTS
C
C                 TITLE                         TITLE AND BORDER THE PLOT
C                   AXIS2                          PLOT AXES
C                      NUMBER                         CALCOMP ROUTINE TO PLOT NUMBERS
C                      PLOT                           CALCOMP ROUTINE TO PLOT POINTS
C                      SYMBOL                         CALCOMP ROUTINE TO PLOT SYMBOLS
C                   NUMBER                         CALCOMP ROUTINE TO PLOT NUMBERS
C                   PLOT                           CALCOMP ROUTINE TO PLOT POINTS
C                   SYMBOL                         CALCOMP ROUTINE TO PLOT SYMBOLS
C
C NOTES
C
C       THIS PROGRAM USES THE CALCOMP HCBS (HOST COMPUTER BASIC SOFTWARE) TO
C       PERFORM ALL PLOTTING.  USERS SHOULD NOTE THAT THIS SOFTWARE VARIES
C       SLIGHTLY FROM VERSION TO VERSION; IN ADDITION, MANY SITES HAVE
C       MODIFIED THIS PACKAGE LOCALLY TO SUIT PARTICULAR NEEDS.
C
C       DOUBLE PRECISION HAS BEEN USED IN THIS PROGRAM TO OBTAIN REQUIRED
C       ACCURACY ON THE VAX 11/780 AND IBM 370 COMPUTERS.  USERS OF CDC CYBER
C       COMPUTERS AND THE LIKE, WHICH USE EXTENDED PRECISION BY DEFAULT,
C       SHOULD NOT REQUIRE DOUBLE PRECISION.
C
C       THIS PROGRAM HAS BEEN EXTENSIVELY MODIFIED ON A VAX 11/780 USING A
C       FORTRAN 77 COMPILER.  EVERY EFFORT HAS BEEN MADE TO USE ANSI 1966
C       STANDARD FORTRAN CONSTRUCTS.  THE ONLY VAX SPECIFIC SUBROUTINE
C       IS 'IOSET'; SEE THE COMMENTS IN THAT ROUTINE FOR MORE SPECIFICS.
C

      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION CONTURD(9),CONTURC(9),EM(6,12),S(6,6)
      DIMENSION C(12),CX(12),CY(12),E(6),EX(6),EY(6)
C
C               VARIOUS GRIDS NEEDED FOR VARIOUS ROUTINES
C
      COMMON /GRDCOM/ CMAT(120,120), CURX(120,120), CURY(120,120),
     1                CURR(120,120), AX(4500), AY(4500)
C
C               COMMON BLOCK PRTCOM IS USED BY SUBROUTINE PRTPRM
C
      COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
     1                MM,NN,NNSKIP
C
C               MAICOM IS USED BY ALL ROUTINES
C
      COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2                S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
C
```

```
C                 RAYCOM IS USED BY ALL ROUTINES REQUIRING RAY DATA
C
      COMMON /RAYCOM/ BDZ,C,CX,CY,D,DELA,DEP,DGDX,DHDX,E,EX,EY,G,GZERO,
     1                HGT,AKFC,AKR,AKS,POT,PREV,P1,P2,P3,P4,P5,QOT,Q1,
     2                Q2,Q3,Q4,Q5,PU,SVV,PDEP,SPREV,GTZERO,
     3                PALFA,SVAV,PG,U,V,IHGT,
     4                NTOREF,INUM,MAXQ,NUMT,NOREF
C
C                 SRFCOM IS USED BY ALL ROUTINES PERFORMING SURFACE FUNCTIONS
C
      COMMON /SRFCOM/ ND,NC,CCON,TTT,ARAY,GR,GT,SARAY,PZ,PZD,
     1                CALFA,PCALFA,PARAY,PREVT,SPREVT,VT,SVT,ASV,
     2                Z,ZD,BZ,KMAX,PX,PY,PTTT,PANGLE,PGR,PPCTD,PPCTCX,
     3                PPCTCY,PGT,SVBZ,SVBDZ,KNUMT,KRFLBU,KRFRBU,CDGDX,
     4                UMX,AGR,PGAM,PBZ,PBDZ,PFK,PV,PVT
C
C                 NUMCOM IS USED BY NUMCON
C
      COMMON /NUMCOM/ CONTURC
C
C                 SET UP THE INPUT FILES AND INITIALIZE THE CALCOMP PLOTTER
C
      CALL IOSET
      CALL PLOTS(0.0,0.0,9)
C
C                 DEFINE SOME CONSTANTS
C
      MMAX=4500
      LI=50
C
C                 READ THE FIRST CONTROL CARD
C
      READ (1,3) MXPLOT,PROJCT,DATE1,DATE2,DIR
C
C                 LOOP OVER THE PLOT SETS
C
      DO 399 NPLOT=1,MXPLOT
         READ (1,4) NOR,NPT,NSK,HT,CIN,NAX,NSH,NCO,NCC,NNSKIP,NXCMAT,MOE
         READ (1,5) MM,NN,CNVRSA,GRID,DCON,DEP,ND,CCON,Z,ZD,NC
C
C                 THE INPUT DIRECTIONS ARE TRANFORMED TO THE COMPUTATIONAL
C                 COORDINATES AND TO RADIANS
C
         ZD=CNVRSA-ZD+180.D0
         ZD=ZD*1.74532925D-2
C
C                 SELECT ACCELERATION DUE TO GRAVITY
C
         IF (MOE .EQ. 0) GO TO 23
         AGR=9.8D0
C
C                 CONVERT TO ENGLISH UNITS FOR PLOT CALCULATIONS
C
```

```
              HT=HT/2.54D0
              GRIDPLT=GRID/0.3048D0
              GO TO 22
 23           AGR=32.2D0
              GRIDPLT=GRID
C
C                    CONVERT CIN FROM SECONDS TO HOURS
C
 22           CIN=CIN/3600.D0
              AMM=MM-1.D0
              ANN=NN-1.D0
              DY=ANN/HT
              SCLI=GRIDPLT*DY*12.
              CALL TITLE (NPLOT,NAX,SCLI,HT)
C
C                    READ THE WATER DEPTH GRID, IF ANY
C
              IF (ND .EQ. 0) GO TO 25
              IF (NXCMAT .NE. 0) GO TO 3939
              READ (2,11) ((CMAT(J,I),I=1,MM),J=1,NN)
C
C                    READ THE CONTURD CARD IF ANY, THEN PLOT SOUNDING DEPTH
C                    CONTOURS REQUESTED
C
 3939         IF (NCO .LE. 0) GO TO 493
              READ (1,495) (CONTURD(I),I=1,NCO)
              CALL NUMCON
C
C                    PLOT THE SHORELINE IF REQUESTED
C
 493          IF (NSH .EQ. 0) GO TO 393
              CALL SHORE
C
C                    READ THE CURRENT GRID, IF ANY
C
 393          IF (NC .EQ. 0) GO TO 3937
 25           IF (NXCMAT .NE. 0) GO TO 3949
              READ(3,11) ((CURY(J,I),I=1,MM),J=1,NN)
              READ(3,11) ((CURX(J,I),I=1,MM),J=1,NN)
C
C                    COMPUTE MAGNITUDE OF CURRENT
C
              DO 50 J=1,NN
                 DO 51 I=1,MM
                    CURR(J,I)=DSQRT(CURX(J,I)**2+CURY(J,I)**2)
 51              CONTINUE
 50           CONTINUE
C
C                    READ THE  CONTURC CARD IF ANY, THEN PLOT CURRENT
C                    PROFILES REQUESTED
C
 3949         IF (NCC .LE. 0) GO TO 3937
              READ(1,495) (CONTURC(I),I=1,NCC)
```

```
             CALL NUMCON2
      C
      C             PRINT SYSTEM PARTICULARS
      C
        3937     CALL PRTPRM(NPLOT)
      C
      C             LOOP OVER THE RAYS OF THIS SET
      C
             DO 15 N=1,NOR
                READ(1,6) DELTAT,TT,X,Y,A,AV,HGTZ,CF,AKRTOL,NROPT,NWBRK,NFAN
      C
      C             SET AND/OR COMPUTE INITIAL VALUES
      C
                TTT=TT
                SDLTAT=DELTAT
                A=CNVRSA-A+180.D0
                A=A*1.74532925D-2
                AV=CNVRSA-AV+180.D0
                AV=AV*1.74532925D-2
                MAXQ=1
                NOLINE=0
                NBRKUP=0
                NFLECT=0
                NRFLBU=0
                NFRACT=0
                NRFRBU=0
                NFLAGR=0
                IFLG=0
                ALFA=0.0D0
                CALFA=0.D0
                SVAV=AV
                ASV=A
                ARAY=A
      C
      C             PERFORM CALCULATIONS FOR THIS RAY
      C
                CALL RAYN(X,Y,A,NPLOT,N,MMAX,LI,AV)
        15       CONTINUE
      C
      C             RE-ORIGIN THE PLOTTER FOR THIS PLOT
      C
             CALL PLOT (-3.,-.4,-3)
      C
      C             THEN CONTINUE WITH THE NEXT PLOT
      C
        399   CONTINUE
      C
      C             CLOSE OUT THE PLOTTER, THEN WRITE TERMINATION RECORD AND END
      C
          CALL PLOT(0,0,999)
          WRITE (6,9999)
          CALL EXIT
      C
```

```
C                 FORMAT SECTION
C
 3      FORMAT(I2,1X,3(A6,1X),A6)
 4      FORMAT(3(2X,I3),5X,2(F8.3,2X),7(2X,I3))
 5      FORMAT(2(2X,I3),1X,F7.3,2(1X,F9.3),1X,F7.2,1X,I3,1X,F9.5,1X,
       1         F7.2,1X,F6.2,1X,I3)
 6      FORMAT(7(F6.2,2X),2(F6.4,2X),3(I1,1X))
 11     FORMAT(16F5.0)
 495    FORMAT(9F8.2)
 9999   FORMAT(1H1,17H THIS IS THE END.)
        END
```

```
        SUBROUTINE TITLE (NPLOT,NAX,SCLI,HT)
C
C         PURPOSE
C
C                 THIS SUBROUTINE LABELS THE PLOT AND ADDS THE STRAIGHT LINE
C                 BORDERS.
C
C         SUBROUTINES REQUIRED
C
C                 AXIS2           SUBROUTINE FOR DRAWING AXES
C                 SYMBOL          CALCOMP ROUTINE FOR DRAWING VARIOUS SYMBOLS
C                 NUMBER          CALCOMP ROUTINE FOR DRAWING NUMBERS
C                 PLOT            CALCOMP ROUTINE FOR DRAWING POINTS AND LINES
C
        IMPLICIT REAL*8 (A-H,O-Z)
        DIMENSION CONTURD(9),EM(6,12),S(6,6)
        COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
       1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
       2                S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
       3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
C
C                 MOVE THE PLOT AXIS, AND DEFINE SOME CONSTANTS
C
        CALL PLOT(3.,0.4,3)
        RT=AMM/DY
        XNPLOT=NPLOT
C
C                 DRAW IN THE PLOT TITLE LABELS
C
        CALL SYMBOL(1.25,0.4,.2,17HPROJ. NO.        ,,90.,17)
        CALL SYMBOL(1.25,2.4,.2,PROJCT,90.,6)
        CALL SYMBOL(1.25,4.0,.2,DATE1,90.,6)
        CALL SYMBOL(1.25,5.2,.2,DATE2,90.,2)
        CALL SYMBOL(1.50,0.4,.2,23HSCL = 1/         , CIN =,90.,23)
        CALL NUMBER(1.50,2.0,.2,SCLI,90.,-1)
        CALL NUMBER(1.50,5.2,.2,CIN*3600.,90.,-1)
        CALL SYMBOL(1.75,0.4,.2,19HPLOT NO.    , DIR. =,90.,19)
        CALL NUMBER(1.75,2.2,.2,XNPLOT,90.,-1)
        CALL SYMBOL(1.75,4.4,.2,DIR,90.,6)
```

```
C
C                 CHECK AXIS TYPE
C
      IF (NAX .NE. 0) GO TO 705
C
C                 DRAW STRAIGHT-LINE BORDERS FOR THE PLOT
C
      CALL PLOT(3.,0.4,3)
      CALL PLOT(3.,HT+.4,2)
      GO TO 706
C
C                 DRAW FULL LABELED AXES
C
  705 CALL AXIS2(3.,0.4,1HY,1,HT,90.,0.,DY)
      CALL AXIS2(3.,.4,1HX,-1,RT,0.,0.,DY)
      CALL PLOT(3.,HT+.4,3)
C
C                 DRAW IN OUTER BOX LIMITS
C
  706 CALL PLOT(RT+3.,HT+.4,2)
      CALL PLOT(RT+3.,.4,2)
C
C                 THEN RE-ORIGIN IN CORRECT POSITION
C
      IF (NAX .EQ. 0) CALL PLOT(3.,0.4,2)
      CALL PLOT(3.,0.4,-3)
      YHT=HT
      RETURN
      END
```

```
      SUBROUTINE AXIS2(X,Y,BCD,NC,SIZE,THETA,YMIN,DY)
C
C       PURPOSE
C
C                 THIS ROUTINE DRAWS, CALIBRATES, AND LABELS THE AXES
C                 FOR THE PLOT.
C
C       SUBROUTINES REQUIRED
C
C                 NUMBER          CALCOMP ROUTINE FOR PLOTTING NUMBERS
C                 PLOT            CALCOMP ROUTINE FOR MOVING THE PEN
C                 SYMBOL          CALCOMP ROUTINE FOR PLOTTING SYMBOLS
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
C                 INITIALIZE VARIOUS CONSTANTS
C
      BIGN=1.0D0
C
C                 IF THERE ARE NO CHARACTERS FOR THE AXIS, THERE IS NO NEED
C                 TO RE-ORIGIN
```

```
C
      IF (NC .GE. 0) GO TO 2
      BIGN=-1.0D0
    2 NAC=IABS(NC)
      TH=THETA*1.74532925D-2
      N=DY*SIZE+0.5D0
      CTH=DCOS(TH)
      STH=DSIN(TH)
      TN=N
      XB=X
      YB=Y
      XA=X-0.1D0*BIGN*STH
      YA=Y+0.1D0*BIGN*CTH
C
C               DRAW AXIS WITH CALIBRATED TICK MARKS
C
      CALL PLOT(XA,YA,3)
      DO 20 I=1,N
         CALL PLOT(XB,YB,2)
         XC=XB+CTH/DY
         YC=YB+STH/DY
         CALL PLOT(XC,YC,2)
         XA=XA+CTH/DY
         YA=YA+STH/DY
         CALL PLOT(XA,YA,2)
         XB=XC
         YB=YC
   20 CONTINUE
      IBSV=YMIN+TN
      XA=XB-(.20D0*BIGN-.05D0)*STH-.02857D0*CTH
      YA=YB+(.20D0*BIGN-.05D0)*CTH-.02857D0*STH
      N=N+1
C
C               NUMBER THE ORIGIN AND EVERY TENTH TICK MARK
C
      DO 30 I=1,N
         IF (MOD(IBSV,10).EQ. 0)
     1                  CALL NUMBER(XA,YA,.1,FLOAT(IBSV),THETA,-1)
         IBSV=IBSV-1
         XA=XA-CTH/DY
         YA=YA-STH/DY
   30 CONTINUE
C
C                LABEL THE AXIS
C
      TNC=NAC+7
      XA=X+(SIZE/2.0-.06*TNC)*CTH-(-.07+BIGN*.36)*STH
      YA=Y+(SIZE/2.0-.06*TNC)*STH+(-.07+BIGN*.36)*CTH
      CALL SYMBOL(XA,YA,.14,BCD,THETA,NAC)
      RETURN
      END
```

```
      SUBROUTINE NUMCON
C
C       PURPOSE
C
C               THIS ROUTINE LOCATES AND DRAWS IN THE SPECIFIED SOUNDING
C               DEPTHS.
C
C       SUBROUTINES REQUIRED
C
C               NUMBER          CALCOMP ROUTINE TO DRAW NUMBERS
C               PLOT            CALCOMP ROUTINE TO MOVE THE PEN
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION CONTURD(9),EM(6,12),S(6,6)
C
      COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2                S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
C
      COMMON /GRDCOM/ CMAT(120,120), CURX(120,120), CURY(120,120),
     1                CURR(120,120), AX(4500), AY(4500)
C
      COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
     1                MM,NN,NNSKIP
C
C               DEFINE SOME USEFUL CONSTANTS
C
      NOD=NN-1
      MODD=MM-1
C
C               SELECT Y-COLUMN STARTING WITH THE SECOND Y-COLUMN
C
      DO 5000 J=2, NOD, NNSKIP
         YJ=J-1
         KKK=1
C
C               SELECT SOUNDING DEPTH
C
         DO 8000 KC=1,NCO
            KWIT=0
            NDIF=3
            I=MM-1
C
C               SEARCH COLUMN FOR THE GIVEN SOUNDING DEPTH BEGINNING ONE
C               GRID UNIT FROM THE END OF THE COLUMN
C
            DO 1011 II=1,MODD
               XI=I-1
               IL=I+1
               XL=IL-1
C
```

```
C                        CHECK FOR CONTOUR REACHING THE SHORELINE
C
                IF (KWIT .GT. 0) GO TO 8000
                IF (CMAT(J,I) .GT. 0) GO TO 20
                KWIT=1
C
C                        LOCATE OURSELVES WITH RESPECT TO THE CONTOUR
C
20              IF (CMAT(J,I)*DCON-CONTURD(KC)) 12,11,13
C
C                        THE CONTOUR FELL ON A GRID POINT
C
11              AX(KKK)=XI
                AY(KKK)=CONTURD(KC)
                KKK=KKK+1
                NDIF=3
                GO TO 1010
C
C                        WE HAVE NOT YET REACHED THE PROPER REGION
C
12              GO TO (14,77,14),NDIF
14              NDIF=1
                GO TO 1010
13              GO TO (77,15,15),NDIF
15              NDIF=2
                GO TO 1010
C
C                        THE CONTOUR IS WITHIN THIS GRID ELEMENT.  LINEARLY
C                        INTERPOLATE FOR THE SOUNDING DEPTH
C
77              SLPX=(DCON*(CMAT(J,IL)-CMAT(J,I)))/(XL-XI)
                XP=(CONTURD(KC)-DCON*CMAT(J,I))/SLPX+XI
                AX(KKK)=XP
                AY(KKK)=CONTURD(KC)
                KKK=KKK+1
                GO TO (81,82),NDIF
81              NDIF=2
                GO TO 1010
82              NDIF=1
1010            I = I - 1
1011         CONTINUE
8000      CONTINUE
C
C                   DRAW OUT SOUNDING DEPTHS FOR EACH SELECTED Y-COLUMN
C
          KKK=KKK-1
          IF (KKK-1) 5000,668,670
670       KKL=KKK-1
          DO 998 IA=1,KKL
             IAD=IA+1
             DO 997 IB=IAD,KKK
                IF (AX(IA) .LE. AX(IB)) GO TO 997
                XMIN=AX(IA)
```

```
                    AX(IA)=AX(IB)
                    AX(IB)=XMIN
                    XMIN=AY(IA)
                    AY(IA)=AY(IB)
                    AY(IB)=XMIN
  997            CONTINUE
  998         CONTINUE
  668         IF (MOD(J,2) .NE. 0) GO TO 104
              KONE=KKK
              KADD=-1
              LAST=1
              GO TO 105
  104         KONE=1
              KADD=1
              LAST=KKK
C
C                     NUMBER THE CONTOUR
C
  105         CALL NUMBER(AX(KONE)/DY,YJ/DY,0.10,AY(KONE),0.0,-1)
              CALL SYMBOL(999.0,999.0,0.10,'D',0.0,1)
              IF (KONE .EQ. LAST) GO TO 5000
              KONE=KONE+KADD
              GO TO 105
  5000    CONTINUE
C
C                     RE-ORIGIN THE PLOTTER AND EXIT
C
          CALL PLOT(0.,0.,-3)
          RETURN
          END
```

```
          SUBROUTINE NUMCON2
C
C            PURPOSE
C
C                     THIS ROUTINE LOCATES AND DRAWS IN THE CURRENT SPEED
C                     CONTOURS.
C
C            SUBROUTINES REQUIRED
C
C                     NUMBER          CALCOMP ROUTINE FOR PLOTTING NUMBERS
C                     PLOT            CALCOMP ROUTINE FOR MOVING THE PEN
C
C
          IMPLICIT REAL*8 (A-H,O-Z)
C
          DIMENSION CONTURD(9),CONTURC(9),EM(6,12),S(6,6),C(12),CX(12)
          DIMENSION CY(12),E(6),EX(6),EY(6)
C
          COMMON /GRDCOM/ CMAT(120,120), CURX(120,120), CURY(120,120),
         1                CURR(120,120), AX(4500), AY(4500)
```

```
C
       COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
     1                 MM,NN,NNSKIP
C
       COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1                 DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2                 S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3                 NOLINE,IFLG:MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
C
       COMMON /RAYCOM/ BDZ,C,CX,CY,D,DELA,DEP,DGDX,DHDX,E,EX,EY,G,GZERO,
     1                 HGT,AKFC,AKR,AKS,POT,PREV,P1,P2,P3,P4,P5,QOT,Q1,
     2                 Q2,Q3,Q4,Q5,PU,SVV,PDEP,SPREV,GTZERO,
     3                 PALFA,SVAV,PG,U,V,IHGT,
     4                 NTOREF,INUM,MAXQ,NUMT,NOREF
C
       COMMON /SRFCOM/ ND,NC,CCON,TTT,ARAY,GR,GT,SARAY,PZ,PZD,
     1                 CALFA,PCALFA,PARAY,PREVT,SPREVT,VT,SVT,ASV,
     2                 Z,ZD,BZ,KMAX,PX,PY,PTTT,PANGLE,PGR,PPCTD,PPCTCX,
     3                 PPCTCY,PGT,SVBZ,SVBDZ,KNUMT,KRFLBU,KRFRBU,CDGDX,
     4                 UMX,AGR,PGAM,PBZ,PBDZ,PFK,PV,PVT
C
       COMMON /NUMCOM/ CONTURC
C
C                DEFINE SOME USEFUL CONSTANTS
C
       NOD=NN-1
       MODD=MM-1
C
C                SELECT Y-COLUMN STARTING WITH THE FOURTH Y-COLUMN
C
       DO 5000 J=4, NOD, NNSKIP
          YJ=J-1
          KKK=1
C
C                SELECT CURRENT SPEED CONTOUR
C
          DO 8000 KC=1,NCC
             NDIF=3
             I=MM-1
C
C                SEARCH COLUMN FOR THE GIVEN CURRENT SPEED CONTOUR BEGINNING
C                ONE GRID UNIT FROM THE END OF THE COLUMN
C
             DO 1011 II=1,MODD
                XI=I-1
                IL=I+1
                XL=IL-1
C
C                    FIGURE OUT WHERE WE ARE IN RESPECT TO THE CONTOUR
C                    LINE
C
 20             IF (CURR(J,I)*CCON-CONTURC(KC)) 12,11,13
C
```

```
C                          THE CONTOUR FELL ON A GRID POINT
C
11                AX(KKK)=XI
                  AY(KKK)=CONTURC(KC)
                  KKK=KKK+1
                  NDIF=3
                  GO TO 1010
12                GO TO (14,77,14),NDIF
14                NDIF=1
                  GO TO 1010
13                GO TO (77,15,15),NDIF
15                NDIF=2
                  GO TO 1010
C
C                          THE CONTOUR FELL WITHIN THE CURRENT GRID ELEMENT.
C                          LINEARLY INTERPOLATE FOR THE CURRENT SPEED CONTOUR
C
77                SLPX=(CCON*(CURR(J,IL)-CURR(J,I)))/(XL-XI)
                  XP=(CONTURC(KC)-CCON*CURR(J,I))/SLPX+XI
                  AX(KKK)=XP
                  AY(KKK)=CONTURC(KC)
                  KKK=KKK+1
                  GO TO (81,82),NDIF
81                NDIF=2
                  GO TO 1010
82                NDIF=1
1010              I=I-1
1011           CONTINUE
8000        CONTINUE
C
C                   DRAW THE SPEED CONTOURS FOR EACH OF THE SELECTED Y-COLUMNS
C
            KKK=KKK-1
            IF (KKK-1) 5000,668,670
670         KKL=KKK-1
            DO 998 IA=1,KKL
               IAD=IA+1
               DO 997 IB=IAD,KKK
                  IF (AX(IA) .LE. AX(IB)) GO TO 997
                  XMIN=AX(IA)
                  AX(IA)=AX(IB)
                  AX(IB)=XMIN
                  XMIN=AY(IA)
                  AY(IA)=AY(IB)
                  AY(IB)=XMIN
997            CONTINUE
998         CONTINUE
668         IF (MOD(J,2) .NE. 0) GO TO 104
            KONE=KKK
            KADD=-1
            LAST=1
            GO TO 105
104         KONE=1
```

```
          KADD=1
          LAST=KKK
C
C                   NUMBER THE CONTOUR
C
105       CALL NUMBER(AX(KONE)/DY,YJ/DY,0.10,AY(KONE),0.0,2)
          CALL SYMBOL(999.0,999.0,0.10,'C',0.0,1)
          IF (KONE .EQ. LAST) GO TO 5000
          KONE=KONE+KADD
          GO TO 105
5000  CONTINUE
C
C                   RESET THE PLOTTER ORIGIN AND EXIT
C
      CALL PLOT(0.,0.,-3)
      RETURN
      END
```

```
      SUBROUTINE SHORE
C
C       PURPOSE
C
C                   THIS ROUTINE IS CALLED TO DRAW IN THE SHORELINE.
C
C       SUBROUTINES REQUIRED
C
C                   PLOT           CALCOMP ROUTINE TO MOVE THE PEN
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION CONTURD(9),EM(6,12),S(6,6)
C
      COMMON /GRDCOM/ CMAT(120,120), CURX(120,120), CURY(120,120),
     1                CURR(120,120), AX(4500), AY(4500)
C
      COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
     1                MM,NN,NNSKIP
C
      COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2                S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
C
C                   INLINE FUNCTION TO LINEARLY INTERPOLATE OVER 2 POINTS
C
      PONT(X1,X2,D1,D2)=X1-D1*((X1-X2)/(D1-D2))
      IC=3
C
C                   SELECT Y-COLUMN
C
      DO 1 J=1,NN
```

```
      YJ=J-1
      JL=J-1
      YL=JL-1
      I=MM
C
C          SEARCH COLUMN FOR ZERO WATER DEPTH STARTING WITH MAXIMUM X
C
      DO 3 II=1,MM
         XI=I-1
         IL=I+1
         XL=IL-1
C
C          FIND OURSELVES WITH RESPECT TO THE SHORELINE
C          (CMAT < 0 => POINT IS ON LAND)
C
         IF (CMAT(J,I)) 100,200,300
C
C          WE ARE ON LAND
C
100      IF (IC .GT. 2) GO TO 102
C
C          INTERPOLATE FOR ZERO WATER DEPTH
C
101      XP=PONT(XI,XL,CMAT(J,I),CMAT(J,IL))
         CALL PLOT(XP/DY,YJ/DY,IC)
         IC=2
         GO TO 1
102      IF (J .LE. 1) GO TO 101
         YP=PONT(YJ,YL,CMAT(J,1),CMAT(JL,1))
         CALL PLOT (0.0,YP/DY,IC)
         IC=2
         XP=PONT(XI,XL,CMAT(J,I),CMAT(J,IL))
         CALL PLOT(XP/DY,YJ/DY,IC)
         GO TO 1
C
C          WE ARE EXACTLY ON THE SHORELINE
C
200      IF (II .NE. MM) GO TO 201
         CALL PLOT(XI/DY,YJ/DY,IC)
         IF (IC .GT. 2) GO TO 204
         IC=3
         GO TO 1
204      IC=2
         GO TO 1
201      IF (IC .LE. 2) GO TO 207
         IF (J .LE. 1) GO TO 207
         YP=PONT(YJ,YL,CMAT(J,1),CMAT(JL,1))
         CALL PLOT (0.0,YP/DY,IC)
         IC=2
207      CALL PLOT(XI/DY,YJ/DY,IC)
         IC=2
         GO TO 1
C
```

```
C                 WE ARE STILL OFF SHORE; KEEP LOOKING FOR LAND
C
  300       IF (II .NE. MM) GO TO 2
            IF (IC .GT. 2) GO TO 1
            YP=PONT(YJ,YL,CMAT(J,1),CMAT(JL,1))
            CALL PLOT (0.0,YP/DY,IC)
            IC=3
            GO TO 1
  2         I=I-1
  3       CONTINUE
  1     CONTINUE
C
C                 RE-ORIGIN THE PLOTTER AND EXIT
C
        CALL PLOT(0.,0.,-3)
        RETURN
        END
```

```
        SUBROUTINE RAYN(X,Y,A,NPLOT,N,MMAX,LI,AV)
C
C         PURPOSE
C
C                 THIS ROUTINE CONTROLS THE CALCULATIONS OF THE WAVE PACKET
C                 PARTICULARS, MOST OF THE PRINTED OUTPUT, AND THE PLOTS OF THE
C                 WAVE PACKET PATHS.
C
C         SUBROUTINES REQUIRED
C
C                 SURFCE          ROUTINE TO COMPUTE RAY PARTICULARS
C                 MOVE            ROUTINE TO MOVE THE WAVE PACKET ALONG ITS PATH
C                 HEIGHT          ROUTINE TO COMPUTE THE WAVE HEIGHT
C                 PCD             ROUTINE TO COMPUTE SOME PERCENT DIFFERENCES
C                 ANGCON          ROUTINE TO CONVERT ANGLES FOR PRINTOUT
C                 PAGCOL          ROUTINE TO PRINT PAGE AND COLUMN HEADINGS
C                 STORE           ROUTINE TO STORE PATH COORDINATES
C                 DRAW            ROUTINE TO DRAW THE RAY PATH
C
        IMPLICIT REAL*8 (A-H,O-Z)
C
        DIMENSION CONTURD(9),EM(6,12),S(6,6)
        DIMENSION C(12),CX(12),CY(12),E(6),EX(6),EY(6)
C
        COMMON /GRDCOM/ CMAT(120,120), CURX(120,120), CURY(120,120),
       1                CURR(120,120), AX(4500), AY(4500)
C
        COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
       1                MM,NN,NNSKIP
C
        COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
       1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
```

```
     3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
C
      COMMON /RAYCOM/ BDZ,C,CX,CY,D,DELA,DEP,DGDX,DHDX,E,EX,EY,G,GZERO,
     1                HGT,AKFC,AKR,AKS,POT,PREV,P1,P2,P3,P4,P5,QOT,Q1,
     2                Q2,Q3,Q4,Q5,PU,SVV,PDEP,SPREV,GTZERO,
     3                PALFA,SVAV,PG,U,V,IHGT,
     4                NTOREF,INUM,MAXQ,NUMT,NOREF
C
      COMMON /SRFCOM/ ND,NC,CCON,TTT,ARAY,GR,GT,SARAY,PZ,PZD,
     1                CALFA,PCALFA,PARAY,PREVT,SPREVT,VT,SVT,ASV,
     2                Z,ZD,BZ,KMAX,PX,PY,PTTT,PANGLE,PGR,PPCTD,PPCTCX,
     3                PPCTCY,PGT,SVBZ,SVBDZ,KNUMT,KRFLBU,KRFRBU,CDGDX,
     4                UMX,AGR,PGAM,PBZ,PBDZ,PFK,PV,PVT
C
C              SET INITIAL VALUES FOR THIS RAY
C
      NDP=1
      NFK=1
      NGO=1
      KREST=0
      KCIN=0
      NOREF=0
      BZ=1.D0
      BDZ=0.D0
      IHGT=1
C
C              CALL SURFCE, MOVE, AND HEIGHT TO DETERMINE VALUES AT
C              THE FIRST RAY POINT
C
      CALL SURFCE(X,Y,A,FK,NFK,NDP,IWAVIT,AV)
      IHGT=0
C
C              SET INITIAL VALUES FOR THIS RAY
C
      INUM=0
      PALFA=ALFA
      SVV=V
      PREV=SVV
      SVT=VT
      GZERO=G
      GTZERO=GT
      TIMEQ=0.
C
C              COMPUTE VALUES AT THE INITIAL RAY POINT
C
      CALL MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP,AV,LI)
      CALL HEIGHT(X,Y,A,FK,NGO,MIT,NFK,NDP,AV)
      GO TO 160
C
C              CHECK FOR AX, AY ARRAY LIMITS EXCEEDED
C
    3 MAXQ=1+MAXQ
      IF (MAXQ+KCIN .LT. MMAX) GO TO 399
```

```
      WRITE (6,401)
      GO TO 190
  399 SAVEGR=GR
C
C                 DETERMINE NEXT RAY POINT
C
      CALL MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP,AV,LI)
      IF (NDP .EQ. 1) GO TO 396
C
C                 RAY REACHED SHORE
C
  402 WRITE (6,403)
      MAXQ=MAXQ-1
      GO TO 190
C
C                 CHECK RETURNED STATUS
C
  396 GO TO (397,397,404,514,515,1900,516,528),MIT
C
C                 PACKET CURVATURE ITERATION NOT CONVERGING
C
  404 WRITE (6,405)
      GO TO 571
C
C                 A CAUSTIC OR FOCAL POINT ENCOUNTERED
C
  514 WRITE (6,504)
      GO TO 571
C
C                 THE WAVE BROKE
C
  515 WRITE (6,505)
      GO TO 571
C
C                 SOME SORT OF REFLECTION HANGUP ENCOUNTERED
C
  516 WRITE (6,517)
      GO TO 571
C
C                 BREAKUP TIME STEP WAS LESS THAN 0.5 SECONDS
C
  528 WRITE (6,529)
  571 MAXQ=MAXQ-1
      GO TO 190
C
C                 COMPUTE TRAVEL TIME ALONG THE RAY
C
  397 TIMEQ=TIMEQ+(D*GRID/(1800.*(GR+SAVEGR)))
  160 IF (ND .NE. 0) CALL PCD(C,E,PCTD)
      IF (NC .EQ. 0) GO TO 614
      CALL PCD(CX,EX,PCTCX)
      CALL PCD(CY,EY,PCTCY)
  614 IF (MAXQ .EQ. 1 .OR. MOD(MAXQ,NSK) .EQ. 0) GO TO 3041
```

```
      GO TO 161
C
C                    WRITE RAY PARTICULARS FOR SELECTED RAY POINTS
C
 3041 ANGLE=A
      CALL ANGCON(ANGLE,CNVRSA)
      GAM=AV
      CALL ANGCON(GAM,CNVRSA)
      ARAY1=ARAY
      CALL ANGCON(ARAY1,CNVRSA)
      ZDP=ZD
      CALL ANGCON(ZDP,CNVRSA)
      IF (MOD(NOLINE,LI) .EQ. 0) CALL PAGCOL(NPLOT,N)
      NOLINE=NOLINE+1
      WRITE (6,612) (MAXQ,X,Y,DEP,Z,ZDP,TTT,ARAY1,
     1    ANGLE,GAM,G,GR,HGT,AKS,AKFC,AKR)
      IF (NPT .EQ. 0) GO TO 161
      ALFAP=ALFA/1.74532925D-2
      CALFAP=CALFA/1.74532925D-2
      IF (MOD(NOLINE,LI) .EQ. 0) CALL PAGCOL(NPLOT,N)
      NOLINE=NOLINE+1
      WRITE(6,615) ALFAP,PCTD,CALFAP,PCTCX,PCTCY,GT,
     1                  U,V,VT,BZ,BDZ,NUMT,FK
      IF (NUMT .LE. 1) GO TO 161
      IF (NRFLBU .EQ. 0) GO TO 616
      WRITE(6,617)
  616 IF (NRFRBU .EQ. 0) GO TO 161
      WRITE(6,618)
C
C                    SAVE PRINTOUT VALUES
C
  161 KRFLBU=NRFLBU
      KRFRBU=NRFRBU
      NRFLBU=0
      NRFRBU=0
      KMAX=MAXQ
      PX=X
      PY=Y
      PDEP=DEP
      PZ=Z
      PZD=ZD
      PTTT=TTT
      PARAY=ARAY
      PANGLE=A
      PGAM=AV
      PGR=GR
      PG=G
      PHGT=HGT
      PKS=AKS
      PKFC=AKFC
      PKR=AKR
      PALFA=ALFA
      IF (NPT .EQ. 0) GO TO 613
```

```
      PPCTD=PCTD
      PCALFA=CALFA
      PPCTCX=PCTCX
      PPCTCY=PCTCY
      PGT=GT
      PU=U
      PV=V
      PVT=VT
      PBZ=BZ
      PBDZ=BDZ
      KNUMT=NUMT
      PFK=FK
C
C                 STORE THE CURRENT RAY POSITION FOR LATER PLOTTING
C
  613 CALL STORE(X,Y,A,KMAX,TIMEQ,KCIN,KREST)
      IF (MIT .EQ. 1) GO TO 10
      IF (MOD(NOLINE,LI) .EQ. 0) CALL PAGCOL(NPLOT,N)
      NOLINE=NOLINE+1
C
C                 PACKET CURVATURE WAS AVERAGED
C
      WRITE (6,9) MAXQ
   10 IF (MAXQ .GT. 1) GO TO 13
      GO TO (3,402),NDP
   13 IF (NGO .EQ. 1) GO TO 3
C
C                 RAY REACHED GRID BOUNDRY
C
      WRITE (6,407)
  190 IF (MAXQ .LE. 1 .OR. MOD(MAXQ,NSK) .EQ. 0) GO TO 1900
C
C                 WRITE RAY PARTICULARS FOR THE LAST POINT
C
      CALL ANGCON (PANGLE,CNVRSA)
      CALL ANGCON (PGAM,CNVRSA)
      CALL ANGCON (PARAY,CNVRSA)
      CALL ANGCON (PZD,CNVRSA)
      IF (MOD(NOLINE,LI) .EQ. 0) CALL PAGCOL(NPLOT,N)
      NOLINE=NOLINE+1
      WRITE (6,612) KMAX,PX,PY,PDEP,PZ,PZD,PTTT,PARAY,PANGLE,PGAM,PG,
     1              PGR,PHGT,PKS,PKFC,PKR
      IF (NPT .EQ. 0) GO TO 1900
      PALFA=PALFA/1.74532925D-2
      PCALFA=PCALFA/1.74532925D-2
      IF (MOD(NOLINE,LI) .EQ. 0) CALL PAGCOL(NPLOT,N)
      WRITE(6,615) (PALFA,PPCTD,PCALFA,PPCTCX,PPCTCY,PGT,
     1   PU,PV,PVT,PBZ,PBDZ,KNUMT,PFK)
      IF (KNUMT .LE. 1) GO TO 1900
      IF (KRFLBU .EQ. 0) GO TO 619
      WRITE(6,617)
  619 IF (KRFRBU .EQ. 0) GO TO 1900
      WRITE(6,618)
```

```
C
C                 DRAW THE RAY PATH
C
1900  CALL DRAW(N,KMAX,KCIN,KREST)
      RETURN
401   FORMAT(80X,35HDIMENSION OF OUTPUT-ARRAYS EXCEEDED)
403   FORMAT(80X,17HRAY REACHED SHORE)
405   FORMAT(80X,41HPACKET CURVATURE ITERATION NOT CONVERGING)
504   FORMAT(80X,22HCAUSTIC OR FOCAL POINT)
505   FORMAT(80X,11HWAVE BREAKS)
517   FORMAT(80X,18HREFLECTION HANG-UP)
529   FORMAT(80X,38HBREAKUP TIME STEP LESS THAN 0.5 SECOND)
612   FORMAT(1X,I5,2F8.2,2(F9.2,F9.4),3F8.2,2F7.2,4F9.4)
615   FORMAT(2X,0PF9.2,F8.2,0PF9.2,2F8.2,4F7.2,F8.4,1X,1PE10.3,9X,I5,
     1        1X,1PE10.3)
617   FORMAT(1H+,93X,7HREFLECT)
618   FORMAT(1H+,96X,4HBETA)
9     FORMAT(80X,4HMMAX=,I4,27H, PACKET CURVATURE AVERAGED)
407   FORMAT(80X,25HRAY REACHED GRID BOUNDARY)
      END




        SUBROUTINE ANGCON(ANG,CNVRSA)
C
C       PURPOSE
C
C                 THIS ROUTINE DEFINES ANGLES WITH RESPECT TO TRUE NORTH
C                 AND PLACES THEM IN THE RANGE 0 TO 360 DEGREES FOR PRINTOUT.
C
C       SUBROUTINES REQUIRED
C
C                 NONE
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
C                 CONVERT THE ANGLE
C
      ANG=ANG/1.74532925D-2
      ANG=CNVRSA-ANG+180.D0
C
C                 FORCE IT INTO 0 -> 360 DEGREES
C
51    IF (ANG .GE. 0.D0) GO TO 50
      ANG=ANG+360.D0
      GO TO 51
50    IF (ANG .LT. 360.D0) GO TO 52
      ANG=ANG-360.D0
      GO TO 50
C
C                 THEN EXIT
C
52    RETURN
```

```
      END
```

```
        SUBROUTINE PAGCOL(NPLOT,N)
C
C       PURPOSE
C
C               THIS ROUTINE PRINTS THE PAGE AND COLUMN HEADINGS AT THE
C               TOP OF A NEW PAGE.
C
C       SUBROUTINES REQUIRED
C
C               NONE
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION CONTURD(9),EM(6,12),S(6,6),C(12),CX(12),CY(12)
      DIMENSION E(6),EX(6),EY(6)
C
      COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
     1                MM,NN,NNSKIP
      COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2                S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
      COMMON /RAYCOM/ BDZ,C,CX,CY,D,DELA,DEP,DGDX,DHDX,E,EX,EY,G,GZERO,
     1                HGT,AKFC,AKR,AKS,POT,PREV,P1,P2,P3,P4,P5,QOT,Q1,
     2                Q2,Q3,Q4,Q5,PU,SVV,PDEP,SPREV,GTZERO,
     3                PALFA,SVAV,PG,U,V,IHGT,
     4                NTOREF,INUM,MAXQ,NUMT,NOREF
C
C               WRITE OUT THE HEADINGS
C
      WRITE (6,7) (PROJCT,DATE1,DATE2,NPLOT,TT,N,DELTAT,CF,AKRTOL)
C
C               CHECK FOR SECOND LINE OF PAGE HEADINGS
C
      IF (MAXQ .NE. 1) GO TO 453
C
C               CHECK FOR METRIC <--> ENGLISH UNITS
C
      IF (MOE .NE. 0) GO TO 465
      WRITE (6,470)
      GO TO 453
465   WRITE (6,471)
C
C               PRINT OUT COLUMN HEADINGS
C
453   WRITE (6,150)
C
C               CHECK FOR USER-REQUESTED EXPANDED PRINTOUT
C
```

```
      IF (NPT .EQ. 0) GO TO 160
      WRITE(6,155)
C
C                DOUBLE SPACE THEN EXIT
C
160   WRITE (6,165)
      RETURN
C
C                FORMAT SECTION
C
7     FORMAT (1H1,12HPROJECT NO. ,A6,1H,,2X,2A6,1H,,5X,8HPLOT NO.,I3,
     1         1H,,1X,7HPERIOD=,F5.1,4HSEC.,1H,,1X,7HRAY NO.,I3,1H,,
     2         1X,7HDELTAT=,F6.2,1H,,1X,3HCF=,F8.6,1H,,1X,
     3         6HKRTOL=,F8.6,//)
150   FORMAT (1X,3HMAX,2X,1HX,7X,1HY,7X,5HDEPTH,4X,
     1    6HCUR:SP,3X,6HCUR:DI,3X,6HPERIOD,3X,
     2    3HRAY,5X,4HPACK,4X,4HWAVE,4X,1HG,6X,2HGR,5X,3HHGT,6X,2HKS,7X,
     3    2HKF,7X,2HKR)
155   FORMAT (2X,7HROTAT:D,2X,5HPCT:D,3X,7HROTAT:C,2X,
     1    6HPCT:CX,2X,6HPCT:CY,2X,2HGT,5X,1HU,6X,1HV,
     2    6X,2HVT,5X,4HBETA,4X,8HDBETA/DT,3X,
     3    6HBRK UP,3X,2HNO,3X,9HCURVATURE)
165   FORMAT (1H0)
470   FORMAT(1X,51HTHE OUTPUT IS IN ENGLISH UNITS.  DEPTH,HGT(FEET).
     1    28HG,GR,GT,U,V,VT(FEET/SECOND).,//)
471   FORMAT(1X,51HTHE OUTPUT IS IN METRIC UNITS.  DEPTH,HGT(METER).
     1    29HG,GR,GT,U,V,VT(METER/SECOND).,//)
      END
```

```
      SUBROUTINE MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP,AV,LI)
C
C       PURPOSE
C
C                THIS SUBROUTINE COMPUTES THE PATH OF THE WAVE PACKET.  TESTS
C                ARE MADE TO LOCATE A REFLECTION POINT AND, IF DESIRED, THE
C                RAY PATH IS CONTINUED BEYOND THE REFLECTION POINT.
C
C       SUBROUTINES REQUIRED
C
C                SURFCE          ROUTINE TO COMPUTE WAVE PARTICULARS
C                ANGCON          ROUTINE TO CONVERT ANGLES FOR PRINTOUT
C                PAGCOL          ROUTINE TO PRINT PAGE AND COLUMN HEADINGS
C                HEIGHT          ROUTINE TO COMPUTE WAVE HEIGHT
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION CONTURD(9),EM(6,12),S(6,6)
      DIMENSION C(12),CX(12),CY(12),E(6),EX(6),EY(6)
C
      COMMON /GRDCOM/ CMAT(120,120), CURX(120,120), CURY(120,120),
     1                CURR(120,120), AX(4500), AY(4500)
```

```
      COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
     1                MM,NN,NNSKIP
      COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2                S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
      COMMON /RAYCOM/ BDZ,C,CX,CY,D,DELA,DEP,DGDX,DHDX,E,EX,EY,G,GZERO,
     1                HGT,AKFC,AKR,AKS,POT,PREV,P1,P2,P3,P4,P5,QOT,Q1,
     2                Q2,Q3,Q4,Q5,PU,SVV,PDEP,SPREV,GTZERO,
     3                PALFA,SVAV,PG,U,V,IHGT,
     4                NTOREF,INUM,MAXQ,NUMT,NOREF
      COMMON /SRFCOM/ ND,NC,CCON,TTT,ARAY,GR,GT,SARAY,PZ,PZD,
     1                CALFA,PCALFA,PARAY,PREVT,SPREVT,VT,SVT,ASV,
     2                Z,ZD,BZ,KMAX,PX,PY,PTTT,PANGLE,PGR,PPCTD,PPCTCX,
     3                PPCTCY,PGT,SVBZ,SVBDZ,KNUMT,KRFLBU,KRFRBU,CDGDX,
     4                UMX,AGR,PGAM,PBZ,PBDZ,PFK,PV,PVT
C
C        INITIALIZE QUANTITIES
C
      AAV=AV
      IWAVIT=1
      IF (ND .EQ. 1 .AND. NC .EQ. 1) IWAVIT=0
      NUMT=1
      MIT=1
      NOREF=0
C
C                SAVE VALUES IN CASE OF BREAKUP OF TIME STEP INTEVAL
C
      SVGR=GR
      SVGT=GT
      SVA=A
      SVAAV=AAV
      SVTTT=TTT
      SVX=X
      SVY=Y
      IF (MAXQ .NE. 2) GO TO 3033
      SVFKB=FK
      GO TO 203
 3033 SVFKB=FKBAR
      SAVFK=FK
  203 IF (MAXQ-2) 38,102,81
  102 FKBAR=FK
C
C                ********************************
C                ********************************
C                **                            **
C                **  BEGIN ITERATION SEGMENT   **
C                **                            **
C                ********************************
C                ********************************
C
C                ITERATE TO FIND VALUES FOR THE NEXT POINT
C
```

```
81    DO 20 IT=1,50
C
C             COMPUTE THE INCREMENTAL DISTANCE TO THE NEXT RAY POINT
C
        D=(GR*DELTAT)/GRID
39      DELA=FKBAR*D
        AA=A+DELA
        ABAR=A+0.5*DELA
        IF (NC .EQ. 1 .OR. DABS(Z) .GT. 1.D-6) GO TO 206
C
C             COMPUTE COMPONENT INCREMENTAL DISTANCES WHEN THERE IS
C             NO CURRENT
C
        DELX=D*DCOS(ABAR)
        DELY=D*DSIN(ABAR)
        ARAY=AA
        GO TO 75
C
C             COMPUTE INCREMENTAL DISTANCES WHEN THERE IS A CURRENT
C
206     DELX=(G*DCOS(ABAR)+Z*DCOS(ZD))*DELTAT/GRID
        DELY=(G*DSIN(ABAR)+Z*DSIN(ZD))*DELTAT/GRID
        ARAY=DATAN2(DELY,DELX)
C
C             DETERMINE THE LOCATION OF THE NEXT POINT
C
75      XX=X+DELX
        YY=Y+DELY
        CALL SURFCE(XX,YY,AA,FKK,NFK,NDP,IWAVIT,AAV)
        AVP=AAV-ALFA
        IF (NTOREF .EQ. 0) GO TO 86
C
C             REFLECTION HAS BEEN DETERMINED ON THE BASIS OF SNELL'S
C             LAW WITH PHASE VELOCITY
C
        NREF=1
        GO TO 13
C
C             DETERMINE IF THE PHASE SPEED IS INCREASING OR DECREASING
C
86      DUD=SVV/PREV
        GO TO (101,6,38,38,38,38,38,38),MIT
101     IF (NDP .EQ. 2) GO TO 38
        FKBAR=0.5*(FK+FKK)
        IF (IT .NE. 49) GO TO 88
        SVFK=FKBAR
88      IF (IT-48) 5,37,9
37      FKKPP=FKBAR
5       IF (MAXQ .GT. 2) GO TO 9
        IF (IT .LE. 1) GO TO 21
C
C             TEST FOR CONVERGENCE OF THE RAY CURVATURE CALCULATIONS
C
```

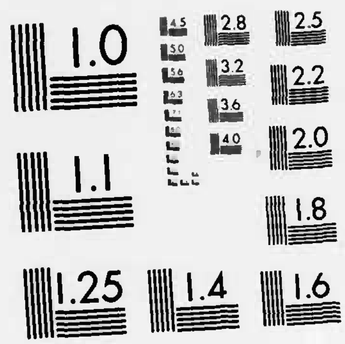
```
9         IF (DABS(FKKP-FKBAR) .LE. 0.00009D0/D .AND.
     1        IWAVIT .EQ. 1                                ) GO TO 6
21        FKKP = FKBAR
20      CONTINUE
C
C                 ******************************
C                 ******************************
C                 **                          **
C                 **  END ITERATION SEGMENT   **
C                 **                          **
C                 ******************************
C                 ******************************
C
C                 DETERMINE IF THE RAY CURVATURE IS CONVERGING TO TWO VALUES
C
        IF (DABS(FKKPP-FKBAR) .LE. 0.00009D0/D .AND. IWAVIT .EQ. 1)
     1    GO TO 18
        IF (ND .EQ. 0) GO TO 161
C
C                 DETERMINE IF CONVERGENCE FAILED DUE TO A REFLECTION POINT
C
        IF (DUD .GT. 1.0 .AND. DABS(DTAN(AVP)) .GT. 5.6712818D0) GO TO 91
161     MIT=3
        GO TO 38
C
C                 REFLECTION IS ASSUMED IF THE PHASE SPEED IS INCREASING AND
C                 THE WAVELET DIRECTION IS WITHIN 10 DEGREES OF BEING PARALLEL
C                 TO THE WATER DEPTH CONTOUR
C
 91     NREF=2
        GO TO 13
 18     FKBAR=.5*(FKBAR+SVFK)
        MIT=2
        GO TO 39
6       IF (ND .EQ. 0) GO TO 92
C
C                 CHECK TO SEE IF WE ARE TOO CLOSE TO A REFLECTION POINT
C
        IF (DUD .LE. 1.0 .OR.
     1    DABS(DTAN(AVP)) .LE. 114.58865D0 .OR.
     2    DABS(DTAN(A-ALFA)) .GE. 3.7320508D0) GO TO 92
C
C                 REFLECTION IS ASSUMED IF THE PHASE SPEED IS INCREASING,
C                 THE WAVELET DIRECTION IS WITHIN 0.5 DEGREES OF BEING
C                 PARALLEL TO THE WATER DEPTH CONTOUR, AND THE WAVE PACKET
C                 DIRECTION IS WITHIN 75 DEGREES OF BEING PERPENDICULAR TO
C                 THE WATER DEPTH CONTOUR
C
        NREF=3
C
C                 **********************************
C                 **********************************
C                 **                              **
```

```
C                 **  BEGIN REFLECTION SEGMENT  **
C                 **                            **
C                 ********************************
C                 ********************************
C
 13     NOREF=NOREF+1
C
C                 TEST FOR REFLECTION HANG-UP
C
        IF (NOREF .LT. 2) GO TO 305
C
C                 MORE THAN ONE REFLECTION HAS OCCURED AT THE SAME POINT
C
        MIT=7
        GO TO 38
C
C                 RECOVER SAVED VALUES FOR PREVIOUS POINT AND SET QUANTITIES
C
 305    DELTAT=SDLTAT
        A=PANGLE
        ASV=A
        SVAV=PGAM
        ALFA=PALFA
        X=PX
        Y=PY
        NUMT=1
        IFLG=0
        INUM=0
        NBRKUP=0
        NFLECT=0
        NRFLBU=0
        NFRACT=0
        NRFRBU=0
        IF (MAXQ .LE. 1 .OR. MOD(MAXQ,NSK) .EQ. 0) GO TO 1900
C
C                 RAY PARTICULARS NEED TO BE WRITTEN FOR THE REFLECTION POINT
C
        CALL ANGCON (PANGLE,CNVRSA)
        CALL ANGCON (PGAM,CNVRSA)
        CALL ANGCON (PARAY,CNVRSA)
        CALL ANGCON (PZD,CNVRSA)
        IF (MOD(NOLINE,LI) .EQ. 0) CALL PAGCOL(NPLOT,N)
        NOLINE=NOLINE+1
        WRITE (6,612) KMAX,PX,PY,PDEP,PZ,PZD,PTTT,
       1              PARAY,PANGLE,PGAM,PG,PGR,HGT,AKS,AKFC,AKR
        IF (NPT .EQ. 0) GO TO 1900
        WALFA=PALFA/1.74532925D-2
        WCALFA=PCALFA/1.74532925D-2
        IF (MOD(NOLINE,LI) .EQ. 0) CALL PAGCOL(NPLOT,N)
        NOLINE=NOLINE+1
        WRITE(6,615) WALFA,PPCTD,WCALFA,PPCTCX,PPCTCY,PGT,
       1             PU,PV,PVT,PRZ,PBDZ,KNUMT,PFK
        IF (KNUMT .LE. 1) GO TO 1900
```

```
      IF (KRFLBU .EQ. 0) GO TO 619
      WRITE(6,617)
619   IF (KRFRBU .EQ. 0) GO TO 1900
      WRITE(6,618)
1900  IF (MOD(NOLINE,LI) .EQ. 0) CALL PAGCOL(NPLOT,N)
C
C                 WRITE OUT TYPE OF REFLECTION
C
3044  GO TO (97,98,99),NREF
97    WRITE (6,152) (KMAX)
      GO TO 300
98    WRITE (6,153) (KMAX)
      GO TO 300
99    WRITE (6,154) (KMAX)
300   NOLINE=NOLINE+1
14    IF (NROPT .NE. 0) GO TO 301
C
C                 RAY IS NOT CONTINUED
C
      MIT=6
      GO TO 38
 301  NTOREF=0
C
C                 COMPUTE REFLECTION ANGLES
C
      SVAV=2.*ALFA-SVAV+3.141592654D0
      A=2.*ALFA-A+3.141592654D0
      ARAY=2.*ALFA-ARAY+3.141592654D0
      AV=SVAV
      AAV=SVAV
      IHGT=1
      CALL SURFCE(X,Y,A,FK,NFK,NDP,IWAVIT,SVAV,CMAT,CURX,CURY,AX,AY)
      IHGT=0
C
C                 SET QUANTITIES FOR REFLECTION POINT
C
      SVV=V
      PREV=SVV
      SVT=VT
      BDZ=-BDZ
      PATI=POT
      QATI=QOT
      NFLAGR=1
      GO TO 102
C
C                 ******************************
C                 ******************************
C                 **                          **
C                 **  END REFLECTION SEGMENT  **
C                 **                          **
C                 ******************************
C                 ******************************
C
```

AD-A134 879 CALCULATION OF WAVE PACKET TRAJECTORIES AND WAVE HEIGHTS FOR VARIABLE WAT..(U) FLORIDA INST OF TECH MELBOURNE DEPT OF OCEANOGRAPHY AND OCEAN.. 2/2

UNCLASSIFIED J E BREEDING ET AL. SEP 82 TR-JEB-11 F/G 8/3 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
C                    DETERMINE IF POINT IS TOO CLOSE TO A GRID BOUNDARY
C
 92   IF ((XX-1.5)*((AMM-1.5)-XX) .GE. 0.0 .AND.
     1    (YY-1.5)*((ANN-1.5)-YY) .GE. 0.0) GO TO 309
      NGO=2
C
C                    UPDATE QUANTITIES AND SAVE VALUES IN CASE OF BREAKUP OF THE
C                    TIME STEP INTERVAL
C
309   XS=XX
      YS=YY
      SVAV=AAV
      IF (ND .EQ. 1 .AND. NC .EQ. 1) IWAVIT=0
      ASV=A
      SPREV=PREV
      PREV=SVV
      SVV=V
      SPREVT=PREVT
      PREVT=SVT
      SVT=VT
      IF (NC .EQ. 0 .AND. DABS(Z) .LT. 1.D-6) AKAY=AA
      AAA=.5*(AA+A)
      A=AA
      AAAV=.5*(AAV+AV)
      AV=AAV
      FK=FKK
      IF (NFLECT .EQ. 1) GO TO 40
C
C                    COMPUTE P AND Q FOR THE INTERMEDIATE POINTS
C
      XX=X+(1.0D0/3.0D0)*DELX*(DABS(DCOS(AAA)))
      YY=Y+(1.0D0/3.0D0)*DELY*(DABS(DSIN(AAA)))
      IHGT=1
      CALL SURFCE(XX,YY,AAA,FKK,NFK,NDP,IWAVIT,AAAV)
      P1=POT
      Q1=QOT
      XX=X+.4*DELX*(DABS(DCOS(AAA)))
      YY=Y+.4*DELY*(DABS(DSIN(AAA)))
      CALL SURFCE(XX,YY,AAA,FKK,NFK,NDP,IWAVIT,AAAV)
      P2=POT
      Q2=QOT
      XX=X+.45573725D0*DELX*(DABS(DCOS(AAA)))
      YY=Y+.45573725D0*DELY*(DABS(DSIN(AAA)))
      CALL SURFCE(XX,YY,AAA,FKK,NFK,NDP,IWAVIT,AAAV)
      P3=POT
      Q3=QOT
      XX=X+(2.D0/3.D0)*DELX*(DABS(DCOS(AAA)))
      YY=Y+(2.D0/3.D0)*DELY*(DABS(DSIN(AAA)))
      CALL SURFCE(XX,YY,AAA,FKK,NFK,NDP,IWAVIT,AAAV)
      P4=POT
      Q4=QOT
      XX=X+.8*DELX*(DABS(DCOS(AAA)))
      YY=Y+.8*DELY*(DABS(DSIN(AAA)))
```

```
      CALL SURFCE(XX,YY,AAA,FKK,NFK,NDP,IWAVIT,AAAV)
      P5=POT
      Q5=QOT
      CALL SURFCE (XS,YS,AA,FKK,NFK,NDP,IWAVIT,AAV)
      IHGT=0
 40   X=XS
      Y=YS
      CALL HEIGHT(X,Y,A,FK,NGO,MIT,NFK,NDP,AV)
      IF (NBRKUP .EQ. 0) GO TO 38
C
C               RECOVER SAVED VALUES BECAUSE OF THE BREAKUP OF THE TIME
C               STEP INTERVAL
C
      IF (IFLG .NE. 0) GO TO 203
      GR=SVGR
      GT=SVGT
      SVV=PREV
      PREV=SPREV
      SVT=PREVT
      PREVT=SPREVT
      A=SVA
      AAV=SVAAV
      AV=SVAAV
      SVAV=SVAAV
      IF (MAXQ .NE. 2) GO TO 311
      FK=SVFKB
      GO TO 312
 311  FKBAR=SVFKB
      FK=SAVFK
 312  TTT=SVTTT
      X=SVX
      Y=SVY
      GO TO 203
   38 RETURN
C
C               FORMAT SECTION
C
 152  FORMAT(1X,5HMAX =,I4,1H,,5X,
     1      43HREFLECTION:  SNELLS LAW WITH PHASE VELOCITY)
 153  FORMAT(1X,5HMAX =,I4,1H,,5X,13HREFLECTION:
     1    41HPACKET CURVATURE ITERATION NOT CONVERGING)
 154  FORMAT(1X,5HMAX =,I4,1H,,5X,34HREFLECTION:  NEAR REFLECTION POINT)
 612  FORMAT (1X,I5,2F8.2,2(F9.2,F9.4),3F8.2,2F7.2,4F9.4)
 615  FORMAT(2X,0PF9.2,F8.2,0PF9.2,2F8.2,4F7.2,F8.4,1X,1PE10.3,9X,I5,
     1      1X,1PE10.3)
 617  FORMAT(1H+,93X,7HREFLECT)
 618  FORMAT(1H+,96X,4HBETA)
      END
```

```
      SUBROUTINE HEIGHT(X,Y,A,FK,NGO,MIT,NFK,NDP,AV)
C
```

```
C        PURPOSE
C
C                THIS ROUTINE COMPUTES THE WAVE HEIGHT.  IF NECESSARY,
C                THE TIME STEP IS SUCCESSIVELY HALVED TO MAINTAIN THE DESIRED
C                ACCURACY IN COMPUTING THE REFRACTION COEFFICIENT, OR THE
C                RAY PATH NEAR A REFLECTION POINT.
C
C        SUBROUTINES REQUIRED
C
C                NONE
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION CONTURD(9),EM(6,12),S(6,6)
      DIMENSION C(12),CX(12),CY(12),E(6),EX(6),EY(6)
C
      COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1               DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2               S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3               NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
      COMMON /RAYCOM/ BDZ,C,CX,CY,D,DELA,DEP,DGDX,DHDX,E,EX,EY,G,GZERO,
     1               HGT,AKFC,AKR,AKS,POT,PREV,P1,P2,P3,P4,P5,QOT,Q1,
     2               Q2,Q3,Q4,Q5,PU,SVV,PDEP,SPREV,GTZERO,
     3               PALFA,SVAV,PG,U,V,IHGT,
     4               NTOREF,INUM,MAXQ,NUMT,NOREF
      COMMON /SRFCOM/ ND,NC,CCON,TTT,ARAY,GR,GT,SARAY,PZ,PZD,
     1               CALFA,PCALFA,PARAY,PREVT,SPREVT,VT,SVT,ASV,
     2               Z,ZD,BZ,KMAX,PX,PY,PTTT,PANGLE,PGR,PPCTD,PPCTCX,
     3               PPCTCY,PGT,SVBZ,SVBDZ,KNUMT,KRFLBU,KRFRBU,CDGDX,
     4               UMX,AGR,PGAM,PBZ,PBDZ,PFK,PV,PVT
C
      IF (MAXQ .GT. 1) GO TO 2
C
C                COMPUTE OR SET QUANTITIES
C
      PATI=POT
      QATI=QOT
      BZTOL=AKRTOL**2
      APQ=1.D-12
      IBDZ=0
      AKF=1.D0
      AKR=1.D0
      AKS=1.D0
      HGT=HGTZ*AKS*AKF*AKR
      AKFC=AKF
      FRICTC=26.318945D0/AGR
      GO TO 38
C
C                COMPUTE SHOALING COEFFICIENT
C
    2 AKS=DSQRT(DABS(GTZERO/GT))
C
C                COMPUTE FRICTION COEFFICIENT
```

```
C
      SKH=6.283185308D0*DEP/(V*TTT)
      IF (NFK .EQ. 2) GO TO 35
      AKF = 1.0D0
      GO TO 36
   35 AKF=1./(AKFC*FRICTC*CF*HGTZ*D*GRID/((TTT**3)*GTZERO)*
     1   (2.*AKS/(DEXP(SKH)-DEXP(-SKH)))**3+1.)
   36 AKFC=AKFC*AKF
C
C                 SAVE VALUES IN CASE OF THE BREAKUP OF THE TIME STEP INTERVAL
C
      SBZ=BZ
      SBDZ=BDZ
C
C                 SET NROPT TO ZERO SO THAT A RAY IS NOT CONTINUED BEYOND A
C                 SECOND REFLECTION POINT
C
      IF (NFLAGR .NE. 0) NROPT=0
      IF (NFK .EQ. 1) GO TO 30
      IF (NFRACT .NE. 0) GO TO 33
      IF (NFLECT .EQ. 0) GO TO 207
      IF (NROPT .EQ. 0 .AND. NFLAGR .EQ. 0) GO TO 33
      GO TO 202
  207 IF (DABS(DTAN(AV-ALFA)) .LE. 5.671282D0 .OR.
     1    DABS(DTAN(A-ALFA)) .GE. 3.7320508D0) GO TO 201
C
C                 IF THE WAVELET DIRECTION IS WITHIN 10 DEGREES OF BEING
C                 PARALLEL TO THE WATER DEPTH CONTOUR AND THE WAVE PACKET
C                 DIRECTION IS WITHIN 75 DEGREES OF BEING PERPENDICULAR TO
C                 THE WATER DEPTH CONTOUR, THE RAY IS ASSUMED TO BE NEAR
C                 A REFLECTION POINT
C
      NFLECT=1
      NRFLBU=1
      IF (NROPT .EQ. 0 .AND. NFLAGR .EQ. 0) GO TO 33
      IF (NFLAGR .NE. 0) IBDZ=1
      GO TO 202
  201 IF (IBDZ .EQ. 0) GO TO 33
      IBDZ=0
C
C                 COMPUTE D(BETA)/DT ANALYTICALLY TO START THE RUNGE KUTTA
C                 CALCULATIONS AFTER THE REFLECTION POINT
C
      BDZ=-(BZ*DSIN(A-ALFA)*DTAN(A-ALFA)*DGDX/GRID)
      IF (NC .EQ. 0) GO TO 205
      BDZC=-(BZ*DSIN(A-CALFA)*DTAN(A-CALFA)*((CDGDX+UMX)/GRID))
      BDZ=BDZ+BDZC
  205 SBDZ=BDZ
C
C                 DETERMINE IF THE WATER DEPTH IS NEARLY CONSTANT
C
   33 IF (DABS(DHDX/GRID) .GT. .00001D0) GO TO 32
   30 IF (DABS(Z) .GT. 1.D-6) GO TO 32
```

```
C
C                   IN DEEP WATER OR WATER OF CONSTANT DEPTH, IF THERE IS NO
C                   CURRENT BETA DOES NOT CHANGE
C
      EBZ=0.D0
      EBDZ=0.D0
      GO TO 71
C
C                   DETERMINE IF P OR Q VALUES ARE NOT ZERO
C
 32   IF (DABS(PATI) .GT. APQ .AND. DABS(P3) .GT. APQ
     1    .AND. DABS(POT) .GT. APQ) GO TO 320
      IF (DABS(QATI) .GT. APQ .AND. DABS(Q3) .GT. APQ
     1    .AND. DABS(QOT) .GT. APQ) GO TO 320
      GO TO 71
C
C                   COMPUTE BETA AND D(BETA)/DT USING THE RUNGE KUTTA METHOD
C
 320  AK1=DELTAT*BDZ
      AL1=-DELTAT*(PATI*BDZ+QATI*BZ)
      AK2=DELTAT*(BDZ+.4*AL1)
      AL2=-DELTAT*(P2*(BDZ+0.4*AL1)+Q2*(BZ+0.4*AK1))
      AK3=DELTAT*(BDZ+2.9697761D-1*AL1+1.5875964D-1*AL2)
      AL3=-DELTAT*(P3*(BDZ+2.9697761D-1*AL1+1.5875964D-1*AL2)+Q3*(BZ+
     1    2.9697761D-1*AK1+1.5875964D-1*AK2))
      AK4=DELTAT*(BDZ+2.1810040D-1*AL1-3.05096516D0*AL2+3.83286476D0
     1    *AL3)
      AL4=-DELTAT*(POT*(BDZ+2.1810040D-1*AL1-3.05096516D0*AL2
     1                   +3.83286476D0*AL3)
     2     +QOT*(BZ+2.1810040D-1*AK1-3.05096516D0*AK2+3.83286476D0*AK3))
      AK5=DELTAT*(BDZ+AL1/3.)
      AL5=-DELTAT*(P1*(BDZ+AL1/3.)+Q1*(BZ+AK1/3.))
      AK6=DELTAT*(BDZ+(6.*AL5+4.*AL1)/25.)
      AL6=-DELTAT*(P2*(BDZ+(6.*AL5+4.*AL1)/25.)+Q2*(BZ+(6.*AK5+4.*AK1)
     1    /25.))
      AK7=DELTAT*(BDZ+(15.*AL6-12.*AL5+AL1)/4.)
      AL7=-DELTAT*(POT*(BDZ+(15.*AL6-12.*AL5+AL1)/4.)+QOT*(BZ+(15.*AK6
     1    -12.*AK5+AK1)/4.))
      AK8=DELTAT*(BDZ+(8.*AL7-50.*AL6+90.*AL5+6.*AL1)/81.)
      AL8=-DELTAT*(P4*(BDZ+(8.*AL7-50.*AL6+90.*AL5+6.*AL1)/81.)+Q4*(BZ+
     1    (8.*AK7-50.*AK6+90.*AK5+6.*AK1)/81.))
      AK9=DELTAT*(BDZ+(8.*AL7+10.*AL6+36.*AL5+6.*AL1)/75.)
      AL9=-DELTAT*(P5*(BDZ+(8.*AL7+10.*AL6+36.*AL5+6.*AL1)/75.)+Q5*(BZ+
     1    (8.*AK7+10.*AK6+36.*AK5+6.*AK1)/75.))
      BZ5=BZ+(1.0D0/192.0D0)*(23.*AK1+125.*AK6-81.*AK8+125.*AK9)
      BDZ5=BDZ+(1.0D0/192.0D0)*(23.*AL1+125.*AL6-81.*AL8+125.*AL9)
      BDZ=BDZ+0.17476028D0*AL1-0.55148066D0*AL2+1.20553560D0*AL3+
     1     0.17118478D0*AL4
      BZ=BZ+0.17476028D0*AK1-0.55148066D0*AK2+1.20553560D0*AK3+
     1   0.17118478D0*AK4
      IF (NC .EQ. 0 .AND. ND .EQ. 1 .AND.
     1    DABS(DTAN(A-ALFA)) .GE. 11.43D0) GO TO 802
      IF (ND .EQ. 0 .AND. NC .EQ. 1 .AND.
```

```
     1    DABS(DTAN(A-CALFA)) .GE. 11.43D0) GO TO 802
      GO TO 801
C
C                 COMPUTE D(BETA)/DT ANALYTICALLY NEAR A CAUSTIC
C
 802  BDZ=-(DSQRT(DABS(QDT)))*SBZ
      BZ5=BZ
      BDZ5=BDZ
C
C                 COMPUTE DIFFERENCE BETWEEN 4TH AND 5TH ORDER SOLUTIONS AS
C                 AN ACCURACY TEST
C
 801  EBZ=BZ-BZ5
      EBDZ=BDZ-BDZ5
C
C                 COMPUTE REFRACTION COEFFICIENT
C
 71   AKR=1./(DSQRT(DABS(BZ)))
      IF (NFLECT .EQ. 0) GO TO 401
 202  IF (IFLG .NE. 0) GO TO 55
C
C                 NEAR A REFLECTION POINT LIMIT THE CHANGE IN THE PACKET
C                 DIRECTION
C
      IF (DABS(DELA) .LT. 1.74532925D-2) GO TO 22
      GO TO 58
  401 IF (IFLG .NE. 0) GO TO 55
C
C                 REQUIRE THAT THE BETA CALCULATION HAS THE DESIRED ACCURACY
C
      IF (DABS(EBZ) .GE. BZTOL .OR. DABS(EBDZ) .GE. BZTOL) GO TO 21
   22 IF (NUMT .LE. 1) GO TO 4
      IFLG=1
      GO TO 55
   21 NFRACT=1
      NRFRBU=1
C
C                 IF THE DESIRED ACCURACY IS NOT REACHED, BREAK UP THE TIME
C                 STEP INTERVAL AND RESUME CALCULATIONS
C
 58   DELTAT=.5*DELTAT
      IF (DELTAT .GE. 0.5) GO TO 81
      MIT=8
      NBRKUP=0
      GO TO 38
 81   NBRKUP=1
C
C                 DOUBLE THE NUMBER OF INTERVALS THE TIME STEP IS DIVIDED INTO
C                 AND RECOVER SAVED VALUES
C
      NUMT=2*NUMT
      BZ=SBZ
      BDZ=SBDZ
```

```
      AKFC=AKFC/AKF
      GO TO 38
   55 INUM=INUM+1
C
C                 DETERMINE IF THE PROPER NUMBER OF POINTS FOR THE BREAKUP OF
C                 THE TIME STEP HAVE BEEN COMPUTED
C
      IF (INUM .LT. NUMT) GO TO 64
C
C                 RESUME CALCULATIONS WITH THE ORIGINAL TIME STEP
C
      IFLG=0
      INUM=0
      DELTAT=SDLTAT
      NBRKUP=0
      GO TO 4
C
C                 UPDATE P AND Q VALUES
C
   64 PATI=POT
      QATI=QOT
C
C                 TEST FOR FOCAL POINT OR CAUSTIC
C
      IF (BZ .GT. 0.0001D0) GO TO 67
   68 MIT=4
      NBRKUP=0
      GO TO 38
   67 NBRKUP=1
      GO TO 38
    4 IF (BZ .LE. 0.0001D0) GO TO 68
C
C                 UPDATE P AND Q VALUES
C
      PATI=POT
      QATI=QOT
      NFRACT=0
      NFLECT=0
C
C                 COMPUTE WAVE HEIGHT
C
      HGT=HGTZ*AKS*AKFC*AKR
      IF (NWBRK .EQ. 0) GO TO 38
C
C                 TEST FOR WAVE BREAK
C
      IF (HGT/(V*TT) .LE. (1./7.)*DTANH(SKH)) GO TO 38
      MIT=5
   38 RETURN
      END
```

```
      SUBROUTINE SURFCE(X,Y,A,FK,NFK,NDP,IWAVIT,AV)
C
C       PURPOSE
C
C               THIS ROUTINE COMPUTES THE WATER DEPTH, CURRENT MAGNITUDE,
C               CURRENT DIRECTION, ROTATION ANGLES, WAVELET DIRECTION,
C               PROPAGATION VELOCITIES, COEFFICIENTS OF THE RAY SEPARATION
C               EQUATION (P AND Q), THE PACKET RAY CURAVATURE, AND THE INITIAL
C               VALUES OF D(BETA)/DT.
C
C       SUBROUTINES REQUIRED
C
C               VELCTY          ROUTINE TO COMPUTE WAVE SPEEDS
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION CONTURD(9),EM(6,12),S(6,6)
      DIMENSION C(12),CX(12),CY(12),E(6),EX(6),EY(6)
      DIMENSION YVW(6)
C
      COMMON /GRDCOM/ CMAT(120,120), CURX(120,120), CURY(120,120),
     1                CURR(120,120), AX(4500), AY(4500)
      COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
     1                MM,NN,NNSKIP
      COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2                S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
      COMMON /RAYCOM/ BDZ,C,CX,CY,D,DELA,DEP,DGDX,DHDX,E,EX,EY,G,GZERO,
     1                HGT,AKFC,AKR,AKS,POT,PREV,P1,P2,P3,P4,P5,QOT,Q1,
     2                Q2,Q3,Q4,Q5,PU,SVV,PDEP,SPREV,GTZERO,
     3                PALFA,SVAV,PG,U,V,IHGT,
     4                NTOREF,INUM,MAXQ,NUMT,NOREF
      COMMON /SRFCOM/ ND,NC,CCON,TTT,ARAY,GR,GT,SARAY,PZ,PZD,
     1                CALFA,PCALFA,PARAY,PREVT,SPREVT,VT,SVT,ASV,
     2                Z,ZD,BZ,KMAX,PX,PY,PTTT,PANGLE,PGR,PPCTD,PPCTCX,
     3                PPCTCY,PGT,SVBZ,SVBDZ,KNUMT,KRFLBU,KRFRBU,CDGDX,
     4                UMX,AGR,PGAM,PBZ,PBDZ,PFK,PV,PVT
C
C               DATA IN THE S AND EM ARRAYS ARE USED IN FITTING QUADRADIC
C               SURFACES FOR WATER DEPTHS AND CURRENTS
C
      DATA S/
     1      2.2248800,-1.5968900,-1.5968900, 0.2500, 0.4736842, 0.2500,
     2     -1.5968900, 1.6895930, 1.0361840,-0.3750,-0.3157895,-0.1875,
     3     -1.5968900, 1.0361840, 1.6895930,-0.1875,-0.3157895,-0.3750,
     4      0.2500000,-0.3750000,-0.1875000, 0.1250, 0.0000000, 0.0625,
     5      0.4736842,-0.3157895,-0.3157895, 0.0000, 0.2105263, 0.0000,
     6      0.2500000,-0.1875000,-0.3750000, 0.0625, 0.0000000, 0.1250/
      DATA   EM/1.,0.,1.,0.,0.,1.,1.,0.,2.,0.,0.,4.,
     1          1.,1.,0.,1.,0.,0.,1.,1.,1.,1.,1.,1.,
     2          1.,1.,2.,1.,2.,4.,1.,1.,3.,1.,3.,9.,
     3          1.,2.,0.,4.,0.,0.,1.,2.,1.,4.,2.,1.,
```

```
     4           1.,2.,2.,4.,4.,4.,1.,2.,3.,4.,6.,9.,
     5           1.,3.,1.,9.,3.,1.,1.,3.,2.,9.,6.,4./
C
      IF (MAXQ .GT. 1) GO TO 201
      IF (ND .EQ. 1) GO TO 573
C
C                IF THERE IS NO WATER DEPTH GRID, THE RAY CURVATURE, P, AND Q
C                DUE TO DEPTH VARIATIONS ARE SET EQUAL TO ZERO
C
      FKAD=0.D0
      POTD=0.D0
      QOTD=0.D0
  573 IF (NC .EQ. 1) GO TO 574
C
C                IF THERE IS NO CURRENT GRID, THE RAY CURAVATURE P AND Q DUE
C                TO CURRENT VARIATIONS ARE SET EQUAL TO ZERO
C
      FKAC=0.D0
      POTC=0.D0
      QOTC=0.D0
  574 IDBDT=0
      IONCE=1
C
C                ***************************************
C                *****                             *****
C                ***** BEGIN SURFACE FITTING SEGMENT *****
C                *****                             *****
C                ***************************************
C
  201 I=X
      J=Y
      FI=I
      FJ=J
      FIC=I
      FJC=J
      XL=X+1.-FI
      YL=Y+1.-FJ
      IF (ND .EQ. 0) GO TO 324
      IF (MAXQ .LE. 1) GO TO 1
      IF (ZI .NE. FI) GO TO 1
      IF (ZJ .EQ. FJ) GO TO 3
    1 ZI=FI
      ZJ=FJ
C
C                SELECT 12 DEPTHS FROM THE GRID ABOUT THE RAY POINT
C
      C(1)=CMAT(J+1,I)
      C(2)=CMAT(J+2,I)
      C(3)=CMAT(J,I+1)
      C(4)=CMAT(J+1,I+1)
      C(5)=CMAT(J+2,I+1)
      C(6)=CMAT(J+3,I+1)
      C(7)=CMAT(J,I+2)
```

```
      C(8)=CMAT(J+1,I+2)
      C(9)=CMAT(J+2,I+2)
      C(10)=CMAT(J+3,I+2)
      C(11)=CMAT(J+1,I+3)
      C(12)=CMAT(J+2,I+3)
C
C              FIT QUADRADIC SURFACE TO THE 12 DEPTHS.  AT SUCCESSIVE RAY
C              POINTS A NEW QUADRADIC SURFACE IS DETERMINED ONLY IF THERE
C              IS A CHANGE IN ANY OF THE 12 WATER DEPTH GRID VALUES
C
      DO 319 II=1,6
         YVW(II)=0.
         DO 318 L=1,12
            YVW(II)=YVW(II)+C(L)*EM(II,L)
  318    CONTINUE
  319 CONTINUE
      DO 321 II=1,6
         E(II)=0.
         DO 320 JJ=1,6
            E(II)=E(II)+S(JJ,II)*YVW(JJ)
  320    CONTINUE
  321 CONTINUE
C
C              COMPUTE INTERPOLATED WATER DEPTH
C
    3 DEP=(E(1)+E(2)*XL+E(3)*YL+E(4)*XL**2+E(5)*XL*YL+E(6)*YL**2)*DCON
C
C              COMPUTE PARTIAL DERIVATIVES OF WATER DEPTH IN FIXED XY-
C              SYSTEM
C
      HX=(E(2)+2.*E(4)*XL+E(5)*YL)*DCON
      HY=(E(3)+E(5)*XL+E(6)*2.*YL)*DCON
      DHDXM=DSQRT((HX*HX)+(HY*HY))
C
C              IF THERE IS A VARIATION IN WATER DEPTH COMPUTE THE ROTATION
C              ANGLE
C
      IF (DABS(DHDXM/GRID) .GT. 0.00001) ALFA=DATAN2(HY,HX)
      COSALF=DCOS(ALFA)
      SINALF=DSIN(ALFA)
C
C              COMPUTE PARTIAL DERIVATIVE OF WATER DEPTH IN ROTATED XY-
C              SYSTEM
C
      DHDX=HX*COSALF+HY*SINALF
      IF (IHGT .EQ. 0) GO TO 572
C
C              COMPUTE SECOND PARTIAL DERIVATIVES OF WATER DEPTH IN FIXED
C              XY-SYSTEM
C
      HXX=2.*E(4)*DCON
      HYY=2.*E(6)*DCON
      HXY=E(5)*DCON
```

```
C
C                 COMPUTE SECOND PARTIAL DERIVATIVES OF WATER DEPTH IN
C                 ROTATED XY-SYSTEM
C
      HDER1=2.*SINALF*COSALF*HXY
      DHDXX=(COSALF**2)*HXX+HDER1+(SINALF**2)*HYY
      DHDYY=(SINALF**2)*HXX-HDER1+(COSALF**2)*HYY
      DHDXY=(SINALF*COSALF)*(HYY-HXX)+((COSALF**2)-(SINALF**2))*HXY
C
C                 IF THE WATER DEPTH IS GREATER THAN 0, THE CALCULATIONS
C                 ARE CONTINUED
C
  572 IF (DEP .GT. 0.) GO TO 324
      NDP=2
      GO TO 403
C
C                 COMPUTE DEEP WATER WAVELENGTH
C
  324 WL=AGR*(TTT**2)/6.283185308D0
      IF (DEP/WL .GT. .64) GO TO 322
      NFK=2
      GO TO 50
C
C                 IN DEEP WATER SET NFK = 1
C
  322 NFK=1
   50 IF (NC .EQ. 0) GO TO 323
      IK=0
      IF (MAXQ .LE. 1) GO TO 52
      IF (ZIC .NE. FIC) GO TO 71
      IF (ZJC .EQ. FJC) GO TO 53
   71 IK=1
   52 ZIC=FIC
      ZJC=FJC
C
C                 SELECT 12 CURRENT SPEEDS FROM X-COMPONENT GRID ABOUT THE RAY
C                 POINT
C
      CX(1)=CURX(J+1,I)
      CX(2)=CURX(J+2,I)
      CX(3)=CURX(J,I+1)
      CX(4)=CURX(J+1,I+1)
      CX(5)=CURX(J+2,I+1)
      CX(6)=CURX(J+3,I+1)
      CX(7)=CURX(J,I+2)
      CX(8)=CURX(J+1,I+2)
      CX(9)=CURX(J+2,I+2)
      CX(10)=CURX(J+3,I+2)
      CX(11)=CURX(J+1,I+3)
      CX(12)=CURX(J+2,I+3)
C
C                 DETERMINE IF THE X-COMPONENT CURRENT VALUES ARE EQUAL
C                 AT THE FOUR NEAREST GRID POINTS
```

```
C
      IF (DABS(CX(5)-CX(8)) .GT. 0.000001D0) GO TO 551
      IF (DABS(CX(4)-CX(9)) .GT. 0.000001D0) GO TO 551
      IF (DABS(CX(5)-CX(9)) .GT. 0.000001D0) GO TO 551
      IF (MAXQ .LE. 1 .OR. IK .EQ. 1) GO TO 562
 53     IF (IZX .EQ. 1) GO TO 561
C
C                 IF THE FOUR GRID POINT VALUES ARE EQUAL THE COMPONENT CURRENT
C                 AND ITS DERIVATIVES ARE SET EQUAL TO ZERO
C
 562  ZX=0.D0
      ZXX=0.D0
      ZXY=0.D0
      ZXXX=0.D0
      ZXYY=0.D0
      ZXXY=0.D0
      IZX=0
      GO TO 557
C
C                 FIT QUADRATIC SURFACE TO THE 12 CURRENT SPEED VALUES FOR THE
C                 X-COMPONENT.  AT SUCCESSIVE RAY POINTS A NEW QUADRATIC
C                 SURFACE IS DETERMINED ONLY IF THERE IS A CHANGE IN ANY OF THE
C                 12 CURRENT SPEED GRID VALUES
C
 551  DO 54 II=1,6
         YVW(II)=0.0D0
         DO 80 L=1,12
            YVW(II)=YVW(II)+CX(L)*EM(II,L)
 80      CONTINUE
 54   CONTINUE
      DO 81 II=1,6
         EX(II)=0.0D0
         DO 55 JJ=1,6
            EX(II)=EX(II)+S(JJ,II)*YVW(JJ)
 55      CONTINUE
 81   CONTINUE
C
C                 COMPUTE INTERPOLATED X-COMPONENT CURRENT
C
 561  ZX=(EX(1)+EX(2)*XL+EX(3)*YL+EX(4)*XL**2+EX(5)*XL*YL+
     1      EX(6)*YL**2)*CCON
C
C                 COMPUTE PARTIAL DERIVATIVES OF X-COMPONENT CURRENT IN
C                 FIXED XY-SYSTEM
C
      ZXX=(EX(2)+2.D0*EX(4)*XL+EX(5)*YL)*CCON
      ZXY=(EX(3)+EX(5)*XL+EX(6)*2.D0*YL)*CCON
      ZXXX=2.D0*EX(4)*CCON
      ZXYY=2.D0*EX(6)*CCON
      ZXXY=EX(5)*CCON
      IZX=1
      IDBDT=1
 557  IF (MAXQ .LE. 1) GO TO 72
```

```
      IF (IK .EQ. 0) GO TO 56
C
C                 SELECT 12 CURRENT SPEEDS FROM Y-COMPONENT GRID ABOUT THE
C                 RAY POINT
C
72    CY(1)=CURY(J+1,I)
      CY(2)=CURY(J+2,I)
      CY(3)=CURY(J,I+1)
      CY(4)=CURY(J+1,I+1)
      CY(5)=CURY(J+2,I+1)
      CY(6)=CURY(J+3,I+1)
      CY(7)=CURY(J,I+2)
      CY(8)=CURY(J+1,I+2)
      CY(9)=CURY(J+2,I+2)
      CY(10)=CURY(J+3,I+2)
      CY(11)=CURY(J+1,I+3)
      CY(12)=CURY(J+2,I+3)
C
C                 DETERMINE IF THE Y-COMPONENT CURRENT VALUES ARE EQUAL
C                 AT THE FOUR NEAREST GRID POINTS
C
      IF (DABS(CY(5)-CY(8)) .GT. 0.000001D0) GO TO 552
      IF (DABS(CY(4)-CY(9)) .GT. 0.000001D0) GO TO 552
      IF (DABS(CY(5)-CY(9)) .GT. 0.000001D0) GO TO 552
      IF (MAXQ .LE. 1 .OR. IK .EQ. 1) GO TO 564
56      IF (IZY .EQ. 1) GO TO 563
C
C                 IF THE FOUR GRID POINT VALUES ARE EQUAL THE COMPONENT CURRENT
C                 AND ITS DERIVATIVES ARE SET EQUAL TO ZERO
C
564   ZY=0.D0
      ZYX=0.D0
      ZYY=0.D0
      ZYXX=0.D0
      ZYYY=0.D0
      ZYXY=0.D0
      IZY=0
      GO TO 553
C
C                 FIT QUADRATIC SURFACE TO THE 12 CURRENT SPEED VALUES FOR
C                 THE Y-COMPONENT.  AT SUCCESSIVE RAY POINTS A NEW QUADRATIC
C                 SURFACE IS DETERMINED ONLY IF THERE IS A CHANGE IN ANY OF
C                 THE 12 CURRENT SPEED GRID VALUES
C
552   DO 57 II=1,6
         YVW(II)=0.D0
         DO 82 L=1,12
            YVW(II)=YVW(II)+CY(L)*EM(II,L)
82       CONTINUE
57    CONTINUE
      DO 59 II=1,6
         EY(II)=0.D0
         DO 58 JJ=1,6
```

```
            EY(II)=EY(II)+S(JJ,II)*YVW(JJ)
58       CONTINUE
59    CONTINUE
C
C                COMPUTE INTERPOLATED Y-COMPONENT CURRENT
C
563   ZY=(EY(1)+EY(2)*XL+EY(3)*YL+EY(4)*XL**2+EY(5)*XL*YL+
     1      EY(6)*YL**2)*CCON
C
C                COMPUTE PARTIAL DERIVATIVES OF Y-COMPONENT CURRENT IN
C                FIXED XY-SYSTEM
C
      ZYX=(EY(2)+2.D0*EY(4)*XL+EY(5)*YL)*CCON
      ZYY=(EY(3)+EY(5)*XL+EY(6)*2.D0*YL)*CCON
      ZYXX=2.D0*EY(4)*CCON
      ZYYY=2.D0*EY(6)*CCON
      ZYXY=EY(5)*CCON
      IZY=1
      IDBDT=1
C
C                DETERMINE CURRENT SPEED MAGNITUDE
C
553   Z=DSQRT((ZX**2)+(ZY**2))
      IF (DABS(Z) .GT. 0.000001D0) GO TO 213
C
C                IF THE CURRENT SPEED IS NEAR ZERO THE CURRENT DERIVATIVES
C                IN THE ROTATED XY-SYSTEM ARE SET EQUAL TO ZERO
C
      DZDX=0.D0
      DZDXX=0.D0
      DZDYY=0.D0
      DZDXY=0.D0
      DZDDX=0.D0
      DZDDY=0.D0
      DZDDXX=0.D0
      DZDDYY=0.D0
      DZDDXY=0.D0
      GO TO 323
C
C                COMPUTE FIRST PARTIAL DERIVATIVES OF CURRENT SPEED IN
C                FIXED XY-SYSTEM
C
213   ZTX=(ZX*ZXX+ZY*ZYX)/Z
      ZTY=(ZX*ZXY+ZY*ZYY)/Z
      DZDXM=DSQRT((ZTX*ZTX)+(ZTY*ZTY))
C
C                IF THERE IS A VARIATION IN CURRENT COMPUTE THE ROTATION ANGLE
C
      IF (DABS(DZDXM/GRID) .GT. 0.00001) CALFA=DATAN2(ZTY,ZTX)
      SICALF=DSIN(CALFA)
      COCALF=DCOS(CALFA)
C
C                COMPUTE FIRST PARTIAL DERIVATIVE OF CURRENT SPEED IN
```

```
C                ROTATED XY-SYSTEM
C
      DZDX=ZTX*COCALF+ZTY*SICALF
C
C                COMPUTE CURRENT DIRECTION
C
      ZD=DATAN2(ZY,ZX)
C
C                COMPUTE FIRST PARTIAL DERIVATIVES OF CURRENT DIRECTION IN
C                FIXED XY-SYSTEM
C
      ZDX=(ZX*ZYX-ZY*ZXX)/(Z**2)
      ZDY=(ZX*ZYY-ZY*ZXY)/(Z**2)
C
C                COMPUTE FIRST PARTIAL DERIVATIVES OF CURRENT DIRECTION IN
C                ROTATED XY-SYSTEM
C
      DZDDX=COCALF*ZDX+SICALF*ZDY
      DZDDY=-SICALF*ZDX+COCALF*ZDY
      IF (IHGT .EQ. 0) GO TO 323
C
C                COMPUTE SECOND PARTIAL DERIVATIVES OF CURRENT SPEED IN FIXED
C                XY-SYSTEM
C
      ZTXX=(ZX*ZXXX+ZXX*ZXX+ZY*ZYXX+ZYX*ZYX-ZTX*ZTX)/Z
      ZTXY=(ZX*ZXXY+ZXX*ZXY+ZY*ZYXY+ZYX*ZYY-ZTX*ZTY)/Z
      ZTYY=(ZX*ZXYY+ZXY*ZXY+ZY*ZYYY+ZYY*ZYY-ZTY*ZTY)/Z
C
C                COMPUTE SECOND PARTIAL DERIVATIVES OF CURRENT SPEED IN ROTATED
C                XY-SYSTEM
C
      ZDER1=2.*SICALF*COCALF*ZTXY
      DZDXX=(COCALF**2)*ZTXX+ZDER1+(SICALF**2)*ZTYY
      DZDYY=(SICALF**2)*ZTXX-ZDER1+(COCALF**2)*ZTYY
      DZDXY=(SICALF*COCALF)*(ZTYY-ZTXX)+((COCALF**2)-(SICALF**2))*ZTXY
C
C                COMPUTE SECOND PARTIAL DERIVATIVES OF CURRENT DIRECTION IN
C                FIXED XY-SYSTEM
C
      ZDXX=(((ZX*ZYXX-ZY*ZXXX)/Z)-2.DO*ZDX*ZTX)/Z
      ZDYY=(((ZX*ZYYY-ZY*ZXYY)/Z)-2.DO*ZDY*ZTY)/Z
      ZDXY=(((ZXX*ZYY+ZX*ZYXY-ZYX*ZXY-ZY*ZXXY)/Z)-2.DO*ZDY*ZTX)/Z
C
C                COMPUTE SECOND PARTIAL DERIVATIVES OF CURRENT DIRECTION IN
C                ROTATED XY-SYSTEM
C
      ZDDER1=2.*SICALF*COCALF*ZDXY
      DZDDXX=(COCALF**2)*ZDXX+ZDDER1+(SICALF**2)*ZDYY
      DZDDYY=(SICALF**2)*ZDXX-ZDDER1+(COCALF**2)*ZDYY
      DZDDXY=(SICALF*COCALF)*(ZDYY-ZDXX)+((COCALF**2)-(SICALF**2))*ZDXY
C
C                ******************************************
C                *****                                *****
```

```
C                  ***** END OF SURFACE FITTING SEGMENT *****
C                  *****                                *****
C                  ******************************************
C
C
C                  COMPUTE VELOCITIES AND DOPPLER SHIFTED WAVE PERIOD
C
  323   DO 60 IT=1,50
           TTO=TTT
           CALL VELCTY(V,TTT,MAXQ,DEP,NFK,U,NC,AGR)
           TTT=TT*(1.D0+Z*DCOS(AV-ZD)/V)
           IF (DABS((TTO-TTT)/TTT) .LE. 0.000001D0) GO TO 61
  60    CONTINUE
  61    IF (IHGT .NE. 0) GO TO 12
C
C                  *********************************************
C                  *****                                   *****
C                  ***** BEGIN WAVELET COMPUTATION SEGMENT *****
C                  *****                                   *****
C                  *********************************************
C
        IF (ND .EQ. 1 .AND. NC .EQ. 1) GO TO 570
        IF (ND .EQ. 0) GO TO 26
C
C                  COMPUTE WAVELET DIRECTION IN ROTATED XY-SYSTEM USING SNELL'S
C                  LAW FOR WATER DEPTHS
C
        GP=SVAV-ALFA
  14    IF (DABS(GP) .LE. 6.283185308D0) GO TO 13
        IF (GP) 16,13,17
  16    GP=GP+6.283185308D0
        GO TO 14
  17    GP=GP-6.283185308D0
        GO TO 14
  13    ARG1=V*DSIN(GP)/SVV
C
C                  TEST FOR TOTAL REFLECTION OF THE WAVELETS
C
        IF (DABS(ARG1) .LE. 1.D0) GO TO 18
        NTOREF=1
        GO TO 403
  18    NTOREF=0
        GPT=DASIN(ARG1)
        IF (DABS(GP) .LE. 4.712388980384D0) GO TO 20
        AVP=6.283185308D0+GPT
        GO TO 22
  20    IF (DABS(GP) .LE. 1.5707963268D0) GO TO 23
        AVP=3.141592654D0-GPT
        GO TO 22
  23    AVP=GPT
  22    AV=AVP+ALFA
        IF (NC .EQ. 0) GO TO 12
C
```

```
C                   COMPUTE WAVELET DIRECTION IN ROTATED XY-SYSTEM USING SNELL'S LAW F
C
26      GPC=SVAV-CALFA
114     IF (DABS(GPC) .LE. 6.283185308D0) GO TO 113
        IF (GPC) 116,113,117
116     GPC=GPC+6.283185308D0
        GO TO 114
117     GPC=GPC-6.283185308D0
        GO TO 114
113     DO 122 ITT=1,50
           AVPSAV=AVP
C
C                   COMPUTE PHASE VELOCITY AND DOPPLER SHIFTED WAVE PERIOD
C
           DO 255 IT=1,50
              TTO=TTT
              CALL VELCTY(V,TTT,MAXQ,DEP,NFK,U,NC,AGR)
              TTT=TT*(1.D0+Z*DCOS(AV-ZD)/V)
              IF (DABS((TTO-TTT)/TTT) .LE. 0.000001D0) GO TO 254
255        CONTINUE
254        VT=V+Z*DCOS(AV-ZD)
           ARG1C=VT*DSIN(GPC)/SVT
C
C                   TEST FOR TOTAL REFLECTION OF THE WAVELETS
C
           IF (DABS(ARG1C) .LE. 1.D0) GO TO 118
           NTOREF=1
           GO TO 403
118        NTOREF=0
           GPCT=DASIN(ARG1C)
           IF (DABS(GPC) .LE. 4.7123889803D0) GO TO 120
           AVP=6.283185308D0+GPCT
           GO TO 115
120        IF (DABS(GPC) .LE. 1.5707963268D0) GO TO 123
           AVP=3.141592654D0-GPCT
           GO TO 115
123        AVP=GPCT
115        AV=AVP+CALFA
           IF (ITT .LE. 1) GO TO 122
           IF (DABS((AVPSAV-AVP)/AVP) .LE. 0.000001D0) GO TO 12
122     CONTINUE
        GO TO 12
C
C                   COMPUTE WAVELET DIRECTION IN ROTATED XY-SYSTEMS USING RAY
C                   CURVATURE EXPRESSION FOR WATER DEPTHS AND CURRENTS
C
570     IF (MAXQ .EQ. 1) GO TO 12
        NTOREF=0
        DELAVS=DELAV
        DELAV=((DTAN(SVAV-ALFA)*DCOS(ARAY-ALFA)*DVDX)+
       1      DCOS(ARAY-CALFA)*(DTAN(SVAV-CALFA)*(CDVDX+UKX)-
       2      (CDVDY+UKY)))*(GR/VT)*(DELTAT/GRID)
        DELAVB=0.5D0*(DELAVS+DELAV)
```

```
      IF (DABS(DELAVB-DELAVS) .LE. 0.000009D0) IWAVIT=1
      AV=SVAV+DELAVB
C
C               *********************************************
C               *****                                   *****
C               ***** END OF WAVELET COMPUTATION SEGMENT *****
C               *****                                   *****
C               *********************************************
C
   12 PHI=A-AV
      COSPHI=DCOS(PHI)
      SINPHI=DSIN(PHI)
      TANPHI=DTAN(PHI)
C
C               COMPUTE GEOMETRIC GROUP VELOCITY
C
      G=U*DCOS(PHI)
      AVMZD=AV-ZD
      SIAVMZ=DSIN(AVMZD)
      COAVMZ=DCOS(AVMZD)
C
C               COMPUTE VELOCITY OF ADVECTED WAVELET FRONT IN CURRENT
C
      VT=V+Z*COAVMZ
      AMZD=A-ZD
      SIAMZD=DSIN(AMZD)
      COAMZD=DCOS(AMZD)
C
C               COMPUTE RAY VELOCITY IN CURRENT
C
      GR=(G**2+Z**2+2*Z*G*COAMZD)**.5
C
C               COMPUTE VELOCITY OF ADVECTED GROUP FRONT IN CURRENT
C
      GT=G+Z*COAMZD
C
C               *******************************************
C               *****                                 *****
C               ***** BEGIN CURVATURE, P AND Q SEGMENT *****
C               *****                                 *****
C               *******************************************
C
C
C
C               COMPUTE ANGULAR FREQUENCY
C
      OMEGA=2.D0*3.141592654D0/TTT
      A2V2=1.0-((OMEGA*V/AGR)**2)
      WHAO=(V**2)+AGR*DEP*A2V2
      OI=2.0*OMEGA*DEP/V
      UVRA=U/V
      UD1=UVRA-0.5
      UD2=(1.0-DSQRT((OI**2)+4.0*(UD1**2)))
```

```
      IF (ND .EQ. 0) GO TO 251
      IF (NFK .EQ. 2) GO TO 402
C
C                IN DEEP WATER THE GEOMETRIC GROUP VELOCITY DERIVATIVE, RAY
C                CURVATURE, P, AND Q FOR WATER DEPTHS ARE SET EQUAL TO ZERO
C
      DVDX=0.D0
      DGDX=0.D0
      FKAD=0.D0
      POTD=0.D0
      QOTD=0.D0
      GO TO 575
  402 W=AGR*V*A2V2/WHAO
C
C                COMPUTE THE FIRST DERIVATIVE OF THE PHASE VELOCITY FOR
C                WATER DEPTHS
C
      DVDX=W*DHDX
      AVMAL=AV-ALFA
      TANAVM=DTAN(AVMAL)
      COSAVM=DCOS(AVMAL)
      AMALFA=A-ALFA
      TANAMA=DTAN(AMALFA)
      COSAMA=DCOS(AMALFA)
      SINAMA=DSIN(AMALFA)
       UD3X=(V/DEP)*DHDX-DVDX
C
C                COMPUTE FIRST DERIVATIVE OF CONVENTIONAL GROUP VELOCITY FOR
C                WATER DEPTHS
C
      DUDX=UVRA*DVDX+UD1*UD2*UD3X
      RHO=1.D0/(1.D0+TANPHI*TANAMA)
      SIGMA=U*SINPHI*TANAVM/V
C
C                COMPUTE FIRST DERIVATIVE OF GEOMETRIC GROUP VELOCITY FOR
C                WATER DEPTHS
C
      DGDX=RHO*(DUDX*COSPHI+SIGMA*DVDX)
C
C                COMPUTE PACKET RAY CURVATURE FOR WATER DEPTHS IN ROTATED
C                XY-SYSTEM
C
      DSRAD=DCOS(ARAY-ALFA)/COSAMA
      FKAD=SINAMA*DGDX*DSRAD
      IF (IHGT .EQ. 0) GO TO 575
C
C                COMPUTE P FOR WATER DEPTHS IN ROTATED XY-SYSTEM
C
      POTD=(-2.*COSAMA*DGDX)/GRID
      DAVDX=TANAVM*DVDX/V
      DADX=TANAMA*DGDX/G
      DPHIDX=DADX-DAVDX
      YH=-2.0*AGR*(V**2)/(WHAO**2)
```

```
C
C                  COMPUTE SECOND DERIVATIVES OF PHASE VELOCITY FOR WATER DEPTHS
C
      DVDXX=W*(DHDXX+YH*(DHDX**2))
      DVDYY=W*DHDYY
      DVDXY=W*DHDXY
      UD4X=(DUDX-UVRA*DVDX)/V
      UD5X=((V*DHDXX-UD3X*DHDX)/DEP)-DVDXX
      UD6=-UD3X*(((OI**2)*UD3X/V)+4.*UD1*UD4X)
      UD7=DSQRT((OI**2)+4.*(UD1**2))
C
C                  COMPUTE SECOND DERIVATIVES OF CONVENTIONAL GROUP VELOCITY
C                  FOR WATER DEPTHS
C
      DUDXX=UVRA*DVDXX+UD4X*(DVDX+UD2*UD3X)+UD1*((UD6/UD7)+UD2*UD5X)
      DUDYY=UVRA*DVDYY+UD1*UD2*((V/DEP)*DHDYY-DVDYY)
      DUDXY=UVRA*DVDXY+UD1*UD2*((V/DEP)*DHDXY-DVDXY)
C
C                  COMPUTE SECOND DERIVATIVES OF GEOMETRIC GROUP VELOCITY FOR
C                  WATER DEPTHS
C
      DGDXX=RHO*(DUDXX*COSPHI-DPHIDX*(2.*DUDX*SINPHI+G*DPHIDX)
     1      -U*SINPHI*(TANAMA*(DADX**2)-TANAVM*((DVDXX/V)+(DAVDX**2))))
      IF (DABS(TANAMA) .GT. 0.08748866D0 .AND. DABS(TANAVM) .GT.
     1    0.08748866D0) GO TO 35
      ZETA=1.
      XI=0.
      GO TO 36
   35 ZETA=1.0/(1.0-TANPHI/TANAMA)
      XI=U*SINPHI/V*TANAVM
   36 DGDYY=ZETA*(DUDYY*COSPHI-XI*DVDYY)
      DGDXY=RHO*(DUDXY*COSPHI+SIGMA*DVDXY)
C
C                  COMPUTE Q FOR WATER DEPTHS IN ROTATED XY-SYSTEM
C
      QOTD=(G*((SINAMA**2)*DGDXX-2.*SINAMA*COSAMA*DGDXY+
     1(COSAMA**2)*DGDYY))/(GRID**2)
  575 IF (NC .EQ. 0) GO TO 252
  251 IF (NFK .EQ. 2) GO TO 203
      WO=-V/OMEGA
      GO TO 204
  203 WO=(DEP*AGR*V*A2V2-(V**3))/(OMEGA*WHAO)
  204 WAVNO=OMEGA/V
      CTAVM=DTAN(AV-CALFA)
C
C                  COMPUTE COMPONENT OF CURRENT NORMAL TO WAVELET FRONT
C
      UK=Z*COAVMZ
      RMKL1=Z*SIAVMZ/VT
      RMK=RMKL1*CTAVM
      RMK1=1.D0+RMK
      RMKL2=UK*(1.D0-WAVNO*WO)/V
      OD1=1.D0/(1.D0-(WAVNO*RMK*WO/RMK1)+RMKL2)
```

```
      OD2=DZDX*COAVMZ+Z*SIAVMZ*DZDDX
      OMEGX=-OD1*(WAVNO/RMK1)*OD2
      RMKY=RMKL1/CTAVM
      RMKY1=1-RMKY
      ODY1=1.D0/(1.D0-(WAVNO*RMKY*WO/RMKY1)+RMKL2)
      ODY2=Z*SIAVMZ*DZDDY
      OMEGY=-ODY1*(WAVNO/RMKY1)*ODY2
C
C               COMPUTE THE FIRST DERIVATIVES OF THE PHASE VELOCITY FOR
C               CURRENTS
C
      CDVDX=WO*OMEGX
      CDVDY=WO*OMEGY
C
C               COMPUTE FIRST DERIVATIVES OF CURRENT COMPONENT NORMAL TO
C               WAVELET FRONT
C
      UKX=(-RMK*CDVDX+OD2)/RMK1
      UKY=(RMKY*CDVDY+ODY2)/RMKY1
      CAMALF=A-CALFA
      CSAM=DSIN(CAMALF)
      CCAM=DCOS(CAMALF)
      CTAM=DTAN(CAMALF)
      IF (NFK .EQ. 2) GO TO 207
C
C               COMPUTE FIRST DERIVATIVES OF CONVENTIONAL GROUP VELOCITY FOR
C               CURRENTS IN DEEP WATER
C
      CDUDX=CDVDX/2.
      CDUDY=CDVDY/2.
      GO TO 208
  207 UDC1=(V/OMEGA)*OMEGX-CDVDX
      UDCY1=(V/OMEGA)*OMEGY-CDVDY
C
C               COMPUTE FIRST DERIVATIVES OF CONVENTIONAL GROUP VELOCITY FOR
C               CURRENTS
C
      CDUDX=UVRA*CDVDX+UD1*UD2*UDC1
      CDUDY=UVRA*CDVDY+UD1*UD2*UDCY1
  208 RMCOM=Z*SIAMZD/GT
      RM=RMCOM*CTAM
      RM1=1.D0+RM
      RNT=(U*SINPHI*CTAM)/GT
      RNG=(U*SINPHI*CTAVM)/VT
      GDC2=DZDX*COAMZD+Z*SIAMZD*DZDDX
      GDC3=RNT/RM1
      RMY=RMCOM/CTAM
      RMY1=1.D0-RMY
      RNTY=(U*SINPHI)/(GT*CTAM)
      RNGY=(U*SINPHI)/(VT*CTAVM)
      GDCY2=Z*SIAMZD*DZDDY
      GDCY3=RNTY/RMY1
C
```

```
C                 COMPUTE FIRST DERIVATIVES OF GEOMETRIC GROUP VELOCITY
C                 FOR CURRENTS
C
      CDGDX=(CDUDX*COSPHI-GDC3*GDC2+RNG*(CDVDX+UKX))/(1.D0+GDC3)
      CDGDY=(CDUDY*COSPHI+GDCY3*GDCY2-RNGY*(CDVDY+UKY))/(1.D0-GDCY3)
C
C                 COMPUTE FIRST DERIVATIVES OF CURRENT COMPONENT NORMAL TO
C                 GROUP FRONT
C
      UMX=(-RM*CDGDX+GDC2)/RM1
      UMY=(RMY*CDGDY+GDCY2)/RMY1
C
C                 COMPUTE PACKET RAY CURVATURE FOR CURRENTS IN ROTATED XY-SYSTEM
C
      DSRAC=DCOS(ARAY-CALFA)/CCAM
      FKAC=(CSAM*(CDGDX+UMX)-CCAM*(CDGDY+UMY))*DSRAC
      IF (IHGT .EQ. 0) GO TO 252
C
C                 COMPUTE P FOR CURRENTS IN ROTATED XY-SYSTEM
C
      POTC=(-2.0*(CCAM*(CDGDX+UMX)+CSAM*(CDGDY+UMY)))/GRID
      IF (NFK .EQ. 2) GO TO 209
      Y0=2.*V/(OMEGA**2)
      GO TO 210
  209 Y01=W0/V
      Y02=(AGR-DEP*(OMEGA**2))*(Y01**2)+2.*DEP*OMEGA*Y01-
     1    (AGR/(OMEGA**2))-DEP
      Y0=(2.*V/WHA0)*(((V/OMEGA)**2)+DEP*A2V2*Y02)
  210 CDAVDX=(CTAVM/VT)*(CDVDX+UKX)
      GMZDX=CDAVDX-DZDDX
      GMZDX2=(GMZDX**2)
      OD3=(DZDXX*COAVMZ-2.D0*DZDX*SIAVMZ*GMZDX-Z*COAVMZ*GMZDX2+
     1     Z*SIAVMZ*(-CTAVM*(CDAVDX**2)+DZDDXX))/RMK1
      OD4=(1.D0-WAVNO*W0)/V
      WAVNOX=OD4*OMEGX
      OD5=1.D0/(1.D0-(WAVNO*RMK*W0/RMK1)+RMKL2)
      OMEGXX=OD5*((WAVNO*RMK*Y0/RMK1)*(OMEGX**2)-WAVNO*OD3-
     1     2.D0*WAVNOX*UKX+(UK*OMEGX/V)*(OD4*CDVDX+
     2     WAVNO*Y0*OMEGX+W0*WAVNOX))
      CDAVDY=-(CDVDY+UKY)/(VT*CTAVM)
      GMZDY=CDAVDY-DZDDY
      GMZDY2=(GMZDY**2)
      ODY3=(DZDYY*COAVMZ-Z*COAVMZ*GMZDY2+
     1     Z*SIAVMZ*(((CDAVDY**2)/CTAVM)+DZDDYY))/RMKY1
      WAVNOY=OD4*OMEGY
      ODY5=1.D0/(1.D0+(WAVNO*RMKY*W0/RMKY1)+RMKL2)
      ODY6=(UK/V)*(OD4*CDVDY+WAVNO*Y0*OMEGY+W0*WAVNOY)
      OMEGYY=ODY5*(-(WAVNO*RMKY*Y0/RMKY1)*(OMEGY**2)-WAVNO*ODY3-
     1     2.D0*WAVNOY*UKY+ODY6*OMEGY)
      ODXY4=(DZDXY*COAVMZ-DZDX*SIAVMZ*GMZDY-Z*COAVMZ*GMZDX*GMZDY+
     1     Z*SIAVMZ*(-((1.D0/CTAVM)+2.D0*CTAVM)*CDAVDX*CDAVDY+
     2     DZDDXY))/RMK1
      OMEGXY=OD5*((WAVNO*RMK*Y0/RMK1)*OMEGX*OMEGY-WAVNO*ODXY4-
```

```
     1     WAVNOY*UKX-WAVNOX*UKY+ODY6*OMEGX)
C
C                COMPUTE SECOND DERIVATIVES OF PHASE VELOCITY FOR CURRENTS
C
      CDVDXX=WO*OMEGXX+YO*(OMEGX**2)
      CDVDYY=WO*OMEGYY+YO*(OMEGY**2)
      CDVDXY=WO*OMEGXY+YO*OMEGX*OMEGY
C
C                COMPUTE SECOND DERIVATIVES OF CURRENT COMPONENT NORMAL TO
C                WAVELET FRONT
C
      UKXX=-(RMK*CDVDXX/RMK1)+OD3
      UKYY=(RMKY*CDVDYY/RMKY1)+ODY3
      UKXY=-(RMK*CDVDXY/RMK1)+ODXY4
      IF (NFK .EQ. 2) GO TO 211
C
C                COMPUTE SECOND DERIVATIVES OF CONVENTIONAL GROUP VELOCITY FOR
C                CURRENTS IN DEEP WATER
C
      CDUDXX=CDVDXX/2.
      CDUDYY=CDVDYY/2.
      CDUDXY=CDVDXY/2.
      GO TO 212
  211 UDC2=(CDUDX-UVRA*CDVDX)/V
      UDC3=((V*OMEGXX-UDC1*OMEGX)/OMEGA)-CDVDXX
      UDC4=-UDC1*(((OI**2)*UDC1/V)+4.*UD1*UDC2)
      UDC5=DSQRT((OI**2)+4.*(UD1**2))
      UDCY2=(CDUDY-UVRA*CDVDY)/V
      UDCY3=((V*OMEGYY-UDCY1*OMEGY)/OMEGA)-CDVDYY
      UDCY4=((OI**2)*UDCY1/V)+4.*UD1*UDCY2
      UDCXY3=((V*OMEGXY-UDCY1*OMEGX)/OMEGA)-CDVDXY
C
C                COMPUTE SECOND DERIVATIVES OF CONVENTIONAL GROUP VELOCITY FOR
C                CURRENTS
C
      CDUDXX=UVRA*CDVDXX+UDC2*(CDVDX+UD2*UDC1)+
     1      UD1*((UDC4/UDC5)+UD2*UDC3)
      CDUDYY=UVRA*CDVDYY+UDCY2*(CDVDY+UD2*UDCY1)+
     1      UD1*((-UDCY1*UDCY4/UDC5)+UD2*UDCY3)
      CDUDXY=UVRA*CDVDXY+UDCY2*(CDVDX+UD2*UDC1)+
     1      UD1*((-UDC1*UDCY4/UDC5)+UD2*UDCXY3)
  212 CDADX=(CTAM/GT)*(CDGDX+UMX)
      CDADY=-(CDGDY+UMY)/(CTAM*GT)
      CDPHDX=CDADX-CDAVDX
      CDPHDY=CDADY-CDAVDY
      TMZDX=CDADX-DZDDX
      TMZDX2=(TMZDX**2)
      TMZDY=CDADY-DZDDY
      TMZDY2=(TMZDY**2)
      GDC4=(COAMZD*DZDXX-2.DO*DZDX*SIAMZD*TMZDX-Z*COAMZD*TMZDX2+
     1     Z*SIAMZD*(-CTAM*(CDADX**2)+DZDDXX))/RM1
      GDCY4=(COAMZD*DZDYY-Z*COAMZD*TMZDY2+
     1     Z*SIAMZD*(((CDADY**2)/CTAM)+DZDDYY))/RMY1
```

```
      RMXY=Z*COAMZD*CTAM/GT
      RMXY1=1.DO+RMXY
      GDCXY4=(COAMZD*DZDXY-TMZDY*(DZDX*SIAMZD+Z*COAMZD*TMZDX)+
     1     Z*COAMZD*(-CDADX*CDADY*((1./CTAM)+2.*CTAM)+
     2     DZDDXY))/RMXY1
C
C              COMPUTE SECOND DERIVATIVES OF GEOMETRIC GROUP VELOCITY FOR
C              CURRENTS
C
      CDGDXX=(CDUDXX*COSPHI-CDPHDX*(2.DO*CDUDX*SINPHI+G*CDPHDX)-
     1     RNT*GDC4-U*SINPHI*CTAM*(CDADX**2)+RNG*(CDVDXX+UKXX)+
     2     U*SINPHI*CTAVM*(CDAVDX**2))/(1.DO+(RNT/RM1))
      CDGDYY=(CDUDYY*COSPHI-CDPHDY*(2.DO*CDUDY*SINPHI+G*CDPHDY)+
     1     RNTY*GDCY4+U*SINPHI*((CDADY**2)/CTAM)-RNGY*(CDVDYY+
     2     UKYY)-U*SINPHI*((CDAVDY**2)/CTAVM))/(1.DO-GDCY3)
      CDGDXY=(CDUDXY*COSPHI-CDPHDY*(CDUDX*SINPHI+G*CDPHDX)-
     1     CDUDY*SINPHI*CDPHDX-RNT*GDCXY4-U*SINPHI*CDADX*
     2     CDADY*((1./CTAM)+2.*CTAM)+RNG*(CDVDXY+UKXY)+
     3     U*SINPHI*CDAVDX*CDAVDY*((1./CTAVM)+2.*CTAVM))/
     4     (1.DO+(RNT/RMXY1))
C
C              COMPUTE SECOND DERIVATIVES OF CURRENT COMPONENT NORMAL TO
C              GROUP FRONT
C
      UMXX=-(RM*CDGDXX/RM1)+GDC4
      UMYY=(RMY*CDGDYY/RMY1)+GDCY4
      UMXY=-(RMXY*CDGDXY/RMXY1)+GDCXY4
C
C              COMPUTE Q FOR CURRENTS IN ROTATED XY-SYSTEM
C
      QOTC=GT*((CSAM**2)*(CDGDXX+UMXX)-2.*CSAM*CCAM*
     1     (CDGDXY+UMXY)+(CCAM**2)*(CDGDYY+UMYY))/(GRID**2)
C
C              COMPUTE PACKET RAY CURVATURE, P, AND Q FOR WATER DEPTHS AND
C              CURRENTS
C
  252 FK=(FKAD+FKAC)/GT
      POT=POTD+POTC
      QOT=QOTD+QOTC
C
C              *********************************************
C              *****                                   *****
C              ***** END OF CURVATURE, P AND Q SEGMENT *****
C              *****                                   *****
C              *********************************************
C
      IF (MAXQ .GT. 1) GO TO 405
C
C              COMPUTE INITIAL D(BETA)/DT FOR WATER DEPTHS
C
      IF (ND .EQ. 1) BDZ=-TANAMA*SINAMA*DGDX/GRID
C
C              COMPUTE INITIAL CHANGE IN WAVELET DIRECTION FOR STARTING THE
```

```
C                 RAY CURVATURE CALCULATIONS OF THE WAVELET DIRECTION
C
      IF (ND .EQ. 1 .AND. NC .EQ. 1) DELAV=
     1     ((DTAN(SVAV-ALFA)*DCOS(ARAY-ALFA)*DVDX)+
     2     DCOS(ARAY-CALFA)*(DTAN(SVAV-CALFA)*(CDVDX+UKX)-
     3     (CDVDY+UKY)))*(GR/VT)*(DELTAT/GRID)
  405 IF (MAXQ .EQ. MAXQSA) GO TO 403
      MAXQSA=MAXQ
      IF (IONCE .EQ. 0 .OR.
     1    IDBDT .EQ. 0 .OR.
     2    DABS(Z) .LE. 1.D-6) GO TO 403
C
C                 COMPUT INITIAL D(BETA)/DT FOR CURRENTS
C
      BDZA=-(BZ*CSAM*CTAM)*((CDGDX+UMX)/GRID)
      BDZ=BDZ+BDZA
      IONCE=0
  403 RETURN
      END
```

```
      SUBROUTINE VELCTY(V,TT,MAXQ,DEP,NFK,U,NC,AGR)
C
C       PURPOSE
C
C                 THIS ROUTINE COMPUTES THE PHASE VELOCITY AND COLINEAR
C                 (CONVENTIONAL) GROUP VELOCITY.
C
C       SUBROUTINES REQUIRED
C
C                 NONE
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      IF (MAXQ .GT. 1 .AND. NC .EQ. 0) GO TO 102
      BAR=6.283185308D0/TT
      CXXO=TT*AGR/6.283185308D0
      CCC=CXXO
      GO TO 103
  102 CCC=XCXY
  103 IF (NFK .EQ. 2) GO TO 105
C
C                 PHASE VELOCITY IS EQUAL TO DEEP WATER VALUE
C
      V=CXXO
      GO TO 106
  105 DO 1000 M=1,90
C
C                 COMPUTE PHASE VELOCITY FOR WATER OF ARBITRARY DEPTH
C
      V=CXXO*DTANH(BAR*DEP/CCC)
      IF (DABS(V-CCC) .LT. .00005D0) GO TO 106
```

```
1000 CCC=(V+CCC)/2.
106  XCXY=V
     BAR2=2.*BAR*DEP/V
     IF (NFK .EQ. 2) GO TO 3036
C
C               COMPUTE DEEP WATER VALUE OF GROUP VELOCITY
C
     U=.5*V
     GO TO 107
C
C               COMPUTE GROUP VELOCITY FOR WATER OF ARBIRTRARY DEPTH
C
3036 U=.5*V*(1.+2.*BAR2/(DEXP(BAR2)-DEXP(-BAR2)))
107  RETURN
     END
```

```
     SUBROUTINE PCD(C,E,PCTDIF)
C
C      PURPOSE
C
C               THIS ROUTINE DETERMINES THE DIFFERENCE BETWEEN THE GRID
C               VALUE AND THE VALUE COMPUTED FROM THE 12-POINT SURFACE
C               FIT FOR THE 4 GRID POINTS CLOSEST TO THE RAY POINT.
C               THE MAXIMUM PERCENTAGE DIFFERENCE OF THE 4 GRID POINTS
C               IS DETERMINED.
C
C      SUBROUTINES REQUIRED
C
C               NONE
C
     IMPLICIT REAL*8 (A-H,O-Z)
C
     DIMENSION C(12),E(6)
     IF (C(4)*C(5)*C(8)*C(9) .NE. 0.) GO TO 901
C
C               PCTDIF IS SET TO 999 IF THE PRODUCT OF THE FOUR GRID
C               VALUES IS ZERO
C
     PCTDIF=999.
     GO TO 902
901  P1=DABS((C(4)-(E(1)+E(2)+E(3)+E(4)+E(5)+E(6)))/C(4))
     P2=DABS((C(5)-(E(1)+E(2)+2.*E(3)+E(4)+E(5)*2.+E(6)*4.))/C(5))
     P3=DABS((C(8)-(E(1)+E(2)*2.+E(3)+E(4)*4.+E(5)*2.+E(6)))/C(8))
     P4=DABS((C(9)-(E(1)+E(2)*2.+E(3)*2.+E(4)*4.+E(5)*4.+E(6)*4.))/
    1   C(9))
     PCTDIF=100.*DMAX1(P1,P2,P3,P4)
902  RETURN
     END
```

```
      SUBROUTINE STORE(X,Y,A,KMAX,TIMEQ,KCIN,KREST)
C
C       PURPOSE
C
C               IN THIS ROUTINE THE COORDINATES OF EACH RAY POINT ARE STORED.
C               IF DESIRED, THE LOCATIONS OF TICK MARKS AT EQUAL TIME
C               INTERVALS ALONG THE RAY ARE COMPUTED AND STORED.
C
C       SUBROUTINES REQUIRED
C
C               NONE
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION CONTURD(9),EM(6,12),S(6,6)
      DIMENSION C(12),CX(12),CY(12),E(6),EX(6),EY(6)
C
      COMMON /GRDCOM/ CMAT(120,120), CURX(120,120), CURY(120,120),
     1                CURR(120,120), AX(4500), AY(4500)
      COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
     1                MM,NN,NNSKIP
      COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2                S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
      COMMON /RAYCOM/ BDZ,C,CX,CY,D,DELA,DEP,DGDX,DHDX,E,EX,EY,G,GZERO,
     1                HGT,AKFC,AKR,AKS,POT,PREV,P1,P2,P3,P4,P5,QOT,Q1,
     2                Q2,Q3,Q4,Q5,PU,SVV,PDEP,SPREV,GTZERO,
     3                PALFA,SVAV,PG,U,V,IHGT,
     4                NTOREF,INUM,MAXQ,NUMT,NOREF
C
      IF (CIN .LE. 0) GO TO 403
      IF (KMAX .GT. 1) GO TO 401
      AT=0.
C
C               STORE POINT COORDINATES
C
  403 KQ=KMAX+KCIN
      AX(KQ)=X
      AY(KQ)=Y
      IF (CIN .LE. 0.) GO TO 205
  402 ZA=A
      SAVEGR=GR
      GO TO 205
  401 ET=TIMEQ-AT
      IF (CIN-ET) 405,404,403
C
C               RAY POINT AND TICK MARK COINCIDE, STORE WITH NEGATIVE X
C
  404 KQ=KMAX+KCIN
      AX(KQ)=-X
      AY(KQ)=Y
      KREST=KREST+1
```

```
      AT=AT+CIN
      GO TO 402
C
C                    COMPUTE LOCATION OF TICK MARK AND STORE WITH NEGATIVE X
C
  405 DSC=(ET-CIN)*(GR+SAVEGR)*3600.D0/(GRID*2.)
      AA=(A+ZA)/2.
      XM=DSC*DCOS(AA)
      YM=DSC*DSIN(AA)
      KQ=KMAX+KCIN
      AX(KQ)=-X+XM
      AY(KQ)=Y-YM
      KREST=KREST+1
      KCIN=KCIN+1
      AT=AT+CIN
      GO TO 401
C
C                    THEN EXIT
C
  205 RETURN
      END
```

```
      SUBROUTINE DRAW (N,KMAX,KCIN,KREST)
C
C         PURPOSE
C
C                    THIS ROTUINE DRAWS AND NUMBERS THE RAYS.  IF DESIRED, TICK
C                    MARKS ARE DRAWN AT EQUAL TIME INTERVALS.
C
C         SUBROUTINES CALLED
C
C                    PLOT             CALCOMP ROUTINE TO MOVE THE PEN
C                    NUMBER           CALCOMP ROUTINE TO DRAW A NUMBER AT A POINT
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION CONTURD(9),EM(6,12),S(6,6)
      DIMENSION C(12),CX(12),CY(12),E(6),EX(6),EY(6)
C
      COMMON /GRDCOM/ CMAT(120,120), CURX(120,120), CURY(120,120),
     1                CURR(120,120), AX(4500), AY(4500)
      COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
     1                MM,NN,NNSKIP
      COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2                S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
      COMMON /RAYCOM/ BDZ,C,CX,CY,D,DELA,DEP,DGDX,DHDX,E,EX,EY,G,GZERO,
     1                HGT,AKFC,AKR,AKS,POT,PREV,P1,P2,P3,P4,P5,QOT,Q1,
     2                Q2,Q3,Q4,Q5,PU,SVV,PDEP,SPREV,GTZERO,
     3                PALFA,SVAV,PG,U,V,IHGT,
```

```
     4                   NTOREF,INUM,MAXQ,NUMT,NOREF
C
      XN=N
      KMAX=KMAX+KCIN
      IF (AX(KMAX) .GE. 0.) GO TO 601
      AX(KMAX)=-AX(KMAX)
      KREST=KREST-1
  601 IF (MOD(N,2) .NE. 0) GO TO 104
C
C                BEGIN EVEN-NUMBERED RAY WITH THE TERMINAL POINT
C
      KTWO=KMAX-1
      KADD=-1
      LAST=1
      MC=KREST+1
      IF (NFAN .EQ. 0) GO TO 201
      CALL NUMBER (AX(KMAX)/DY,AY(KMAX)/DY,0.1,XN,0.0,-1)
  201 CALL PLOT (AX(KMAX)/DY,AY(KMAX)/DY,3)
      IF (KMAX .LE. 1) GO TO 106
      GO TO 105
C
C                BEGIN ODD NUMBERED RAY WITH THE INITIAL POINT
C
  104 KTWO=2
      KADD=1
      LAST=KMAX
      MC=0
      IF (NFAN .NE. 0) GO TO 111
C
C                NUMBER RAY AT THE INITIAL POINT
C
      CALL NUMBER (AX(1)/DY,AY(1)/DY,0.1,XN,0.0,-1)
  111 CALL PLOT (AX(1)/DY,AY(1)/DY,3)
      IF (KMAX .LE. 1) GO TO 106
  105 IF (CIN .LE. 0.) GO TO 300
      IF (AX(KTWO) .LT. 0.) GO TO 302
C
C                DRAW SEGMENT OF RAY
C
  300 CALL PLOT (AX(KTWO)/DY,AY(KTWO)/DY,2)
      GO TO 303
  302 AX(KTWO)=-AX(KTWO)
      WI=.05
      MC=MC+KADD
      IF (MOD(MC,10) .NE. 0) GO TO 500
      WI=.10
  500 XPN=AX(KTWO)/DY
      YPN=AY(KTWO)/DY
      KQ=KTWO-KADD
  430 XPL=AX(KQ)/DY
      YPL=AY(KQ)/DY
      IF (DABS(XPN-XPL) .LT. .0005D0 .AND.
     1    DABS(YPN-YPL) .LT. .0005D0) GO TO 410
```

```
      GO TO 420
C
C                 POINTS ARE TOO CLOSE TOGETHER
C
  410 KQ=KQ-KADD
      GO TO 430
  420 DSC=DSQRT((XPN-XPL)**2+(YPN-YPL)**2)
C
C                 DRAW THE RAY
C
      CALL PLOT(XPN,YPN,2)
      XB=WI*(YPN-YPL)/DSC
      YB=-WI*(XPN-XPL)/DSC
C
C                 DRAW THE TICK MARK ON THE RAY
C
      CALL PLOT (XPN+XB,YPN+YB,2)
      CALL PLOT (XPN-XB,YPN-YB,2)
      CALL PLOT (XPN,YPN,2)
  303 IF (KTWO .EQ. LAST) GO TO 106
      KTWO=KTWO+KADD
      GO TO 105
  106 IF (KADD .GE. 0) GO TO 108
      IF (NFAN .NE. 0) GO TO 205
      CALL NUMBER (AX(1)/DY,AY(1)/DY,0.1,XN,0.0,-1)
      GO TO 205
  108 IF (NFAN .EQ. 0) GO TO 205
C
C                 NUMBER THE RAY AT THE TERMINAL POINT
C
      CALL NUMBER (AX(KMAX)/DY,AY(KMAX)/DY,0.1,XN,0.0,-1)
  205 RETURN
      END
```

```
        SUBROUTINE IOSET
C
C       PURPOSE
C
C               THIS SUBROUTINE SETS UP THE I/O FILES FOR THE VARIOUS
C               WAVPAK ROUTINES.  THE USER IS ASKED FOR EACH FILE NAME,
C               WHICH IS THEN OPENED AND MADE READY FOR I/O.  THIS IS
C               THE ONLY TOTALLY VAX DEPENDENT SUBROUTINE, AND MAY BE
C               REPLACED BY THE APPROPRIATE JOB CONTROL LANGUAGE ASSIGNMENTS
C               FOR THE PARTICULAR MACHINE USED.
C
C       SUBROUTINES REQUIRED
C
C               NONE
C
        CHARACTER*20 FILE_NAME
C
```

```
C                 FIRST, SET UP THE DATA INPUT FILE NAME
C
        TYPE 100,'$ENTER THE CONTROL FILE NAME '
        READ(5,100) FILE_NAME
        CLOSE(UNIT=1)
        OPEN(UNIT=1,NAME=FILE_NAME,TYPE='OLD',DEFAULTFILE='.CTL')
C
C                 NOW THE WATER DEPTH GRID INPUT FILE
C
        TYPE 100,'$ENTER WATER DEPTH GRID FILE NAME <NONE> '
        READ(5,100) FILE_NAME
        IF (FILE_NAME .EQ. ' ') GOTO 20
        CLOSE(UNIT=2)
        OPEN(UNIT=2,NAME=FILE_NAME,TYPE='OLD',DEFAULTFILE='.GRD')
C
C                  NOW THE CURRENT SPEED INPUT FILE
C
 20     CONTINUE
        TYPE 100,'$ENTER CURRENT SPEED GRID FILE NAME <NONE> '
        READ(5,100) FILE_NAME
        IF (FILE_NAME .EQ. ' ') GOTO 30
        CLOSE(UNIT=3)
        OPEN(UNIT=3,NAME=FILE_NAME,TYPE='OLD',DEFAULTFILE='.GRD')
C
C                  NOW THE OUTPUT FILE NAME
C
 30     TYPE 100,'$ENTER THE OUTPUT FILE NAME '
        READ(5,100) FILE_NAME
        CLOSE(UNIT=6)
        OPEN(UNIT=6,NAME=FILE_NAME,TYPE='NEW',DEFAULTFILE='.OUT')
C
C                 NOW THE PLOT FILE NAME
C
        TYPE 100,'$ENTER THE PLOT FILE NAME <NONE> '
        READ(5,100) FILE_NAME
        IF (FILE_NAME .NE. ' ') GOTO 10
        CLOSE(UNIT=9)
        OPEN(UNIT=9,NAME='NL:DUMMY.PLT',STATUS='NEW',
        1    CARRIAGECONTROL='LIST')
        RETURN
 10     CLOSE(UNIT=9)
        OPEN(UNIT=9,NAME=FILE_NAME,STATUS='NEW',CARRIAGECONTROL='LIST',
        1    DEFAULTFILE='.PLT')
        RETURN
 100    FORMAT(A)
        END
```

```
      SUBROUTINE PRTPRM(NPLOT)
C
C       PURPOSE
C
```

```
C                 THIS SUBROUTINE PRINTS THE PARAMETERS READ FROM THE
C                 CONTROL FILE.  THE RAY PARAMETERS ARE NOT READ AT THIS
C                 TIME.
C
C          SUBROUTINES REQUIRED
C
C                 NONE
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION CONTURD(9),CONTURC(9),EM(6,12),S(6,6),CONTURC1(9)
      DIMENSION C(12),E(6),EX(6),EY(6),CONTURD1(9),CX(12),CY(12)
C
      COMMON /PRTCOM/ HT,MXPLOT,NOR,NPT,NAX,NSH,NCO,NCC,NXCMAT,
     1                MM,NN,NNSKIP
C
      COMMON /MAICOM/ ALFA,AMM,ANN,CF,CIN,CNVRSA,CONTURD,DATE1,DATE2,
     1                DCON,DELTAT,DIR,DY,EM,GRID,HGTZ,AKRTOL,PROJCT,
     2                S,SDLTAT,NSK,TT,NWBRK,NBRKUP,NFAN,NFLAGR,
     3                NOLINE,IFLG,MOE,NFLECT,NFRACT,NRFLBU,NRFRBU,NROPT
C
      COMMON /RAYCOM/ BDZ,C,CX,CY,D,DELA,DEP,DGDX,DHDX,E,EX,EY,G,GZERO,
     1                HGT,AKFC,AKR,AKS,POT,PREV,P1,P2,P3,P4,P5,QOT,Q1,
     2                Q2,Q3,Q4,Q5,PU,SVV,PDEP,SPREV,GTZERO,
     3                PALFA,SVAV,PG,U,V,IHGT,
     4                NTOREF,INUM,MAXQ,NUMT,NOREF
C
      COMMON /SRFCOM/ ND,NC,CCON,TTT,ARAY,GR,GT,SARAY,PZ,PZD,
     1                CALFA,PCALFA,PARAY,PREVT,SPREVT,VT,SVT,ASV,
     2                Z,ZD,BZ,KMAX,PX,PY,PTTT,PANGLE,PGR,PPCTD,PPCTCX,
     3                PPCTCY,PGT,SVBZ,SVBDZ,KNUMT,KRFLBU,KRFRBU,CDGDX,
     4                UMX,AGR,PGAM,PBZ,PBDZ,PFK,PV,PVT
C
      COMMON/NUMCOM/CONTURC
C
C                 SOME LOCAL THINGS
C
      REAL*4 YES,NO,YESNO,INCH,CENTIM,INCM
C
      DATA YES,NO,INCH,CENTIM/'YES ','NO  ',' IN ',' CM '/
C
      REAL*8 METRIC,ENGLIS,METENG,FEET,METERS,FETMET
C
      DATA METRIC,ENGLIS,FEET,METERS/'METRIC  ','ENGLISH ',' FEET   ',
     1                               ' METERS '/
C
C                 SET UP UNIT IDENTIFIERS.  DEFAULT IS ENGLISH
C
      INCM = INCH
      METENG = ENGLISH
      FETMET = FEET
      DCON1 = DCON
      GRID1 = GRID
```

```
      CIN1 = CIN
      IF (MOE .EQ. 0) GO TO 10
C
C                 MOE <> 0 => UNITS ARE METRIC
C
      INCM = CENTIM
      METENG = METRIC
      FETMET = METERS
      DCON1=DCON*0.3048
      GRID1=GRID*0.3048D0
      CIN1=CIN*3600D0
      IF (ND .EQ. 0) GO TO 20
      DO 4000 I=1,NCO
         CONTURD1(I)=CONTURD(I)/DCON
 4000 CONTINUE
 20   IF (NC .EQ. 0) GO TO 10
      DO 3000 I=1,NCC
         CONTURC1(I)=CONTURC(I)/CCON
 3000 CONTINUE
C
C                 FIRST SET OF DATA
C
 10     WRITE(6,100) NPLOT,PROJCT,DATE1,DATE2,DIR,MXPLOT,METENG
C
C                 THE MEANING OF NPT HAS BEEN CHANGED.
C
C
C                 SECOND SET.  IF NPT = 0, THEN DON'T PRINT INTERMID. RAY DATA
C
      YESNO = YES
      IF (NPT .EQ. 0) YESNO = NO
      WRITE(6,200) NOR,YESNO,NSK,CIN1,HT,INCH
C
C                 THIRD SET.  NAX = 0 => NO CALIBRATE AXIS
C
      YESNO = YES
      IF (NAX .EQ. 0) YESNO = NO
      WRITE(6,300) YESNO
C
C                 FOURTH SET. NSH = 0 => DON'T PLOT SHORELINE
C
      YESNO = YES
      IF (NSH .EQ. 0) YESNO = NO
      WRITE(6,400) YESNO
C
C                 FIFTH SET.
C
      IF (ND .EQ. 0) GO TO 30
      WRITE(6,500) NCO
 30   IF (NC .EQ. 0) GO TO 40
      WRITE(6,510) NCC
C
C                 SIXTH SET.  IF NXCMAT = 0 THEN READ A NEW CMATRIX
```

```
C
40     YESNO = NO
       IF (NXCMAT .EQ. 0) YESNO = YES
       WRITE(6,600) YESNO
       IF (NXCMAT .EQ. 0) WRITE(6,610) MM,NN,GRID1,FETMET
C
C                SEVENTH SET.
C
       WRITE(6,700) CNVRSA,DCON1,CCON,NNSKIP
C
C                EIGTH SET
C
       IF (ND .EQ. 0) WRITE(6,710) DEP
       IF (ND .NE. 0) WRITE(6,720)
       ZDPRIM = (CNVRSA + 180.0) - (ZD/1.74532925D-2)
       IF (NC .EQ. 0) WRITE(6,730) Z,ZDPRIM
       IF (NC .NE. 0) WRITE(6,740)
C
C                LAST SET -- CONTURS IF NEEDED
C
       IF (NCC .NE. 0) WRITE(6,920) METENG,(CONTURC1(I),I=1,NCC)
       IF (NCD .NE. 0) WRITE(6,930) METENG,(CONTURD1(I),I=1,NCD)
C
C                AND SPLIT BACK TO THE SALT MINES
C
       RETURN
100    FORMAT('1WAVPAK -- WAVE PACKET TRAJECTORY ANALYSIS PROGRAM V2.0'//
      1' SYSTEM PARAMTERS FOR PLOT NUMBER ',I2,':'//
      2'      PROJECT NAME ............................. ',A6/
      3'      PROJECT DATE ............................. ',2A6/
      4'      PLOT IDENTIFICATION LABEL ................ ',A6/
      5'      NUMBER OF RUNS TO BE MADE ................ ',I3//
      6'      DATA UNITS ............................... ',A8)
200    FORMAT('      NUMBER OF RAYS TO BE RUN ................. ',I3/
      1'      PRINT INTERMEDIATE RAY RESULTS ............ ',A4/
      2'      INTERMEDIATE RAY RESULT INTERVAL .......... ',I3/
      3'      RAY TICK MARK SPACING ..................... ',F8.3,' SEC'/
      4'      PLOTTER Y-AXIS LENGTH ..................... ',F8.3,A4)
300    FORMAT('      CALIBRATE AND LABEL AXES ................. ',A4)
400    FORMAT('      PLOT SHORELINE ........................... ',A4)
500    FORMAT('      NUMBER OF SOUNDING DEPTHS ................ ',I3)
510    FORMAT('      NUMBER OF CURRENT SOUNDINGS............... ',I3)
600    FORMAT('      READ A NEW GRID .......................... ',A4)
610    FORMAT('      GRID DIMENSIONS .......................... ('
      1                                              ,I3,',',I3,')'/
      2'      GRID SPACING .............................. ',1PE10.3,A8)
700    FORMAT('      RAY CONVERSION ANGLE ..................... ',F8.3/
      1'      WATER DEPTH CONVERSION FACTOR.............. ',1PE10.3/
      2'      CURRENT SPEED CONVERSION FACTOR ........... ',1PE10.3/
      3'      SOUNDING  INCREMENT ....................... ',I3)
710    FORMAT('      WATER DEPTH GRID SPECIFIED ............... NO'/
      1'        WATER DEPTH CONSTANT .................. ',1PE10.3)
720    FORMAT('      WATER DEPTH GRID SPECIFIED ............... YES')
```

```
730   FORMAT('       CURRENT GRID SPECIFIED .................... NO'/
     1'           INITIAL CURRENT SPEED .................. ',1PE10.3/
     2'           INITIAL CURRENT DIRECTION .............. ',1PE10.3)
740   FORMAT('       CURRENT GRID SPECIFIED .................... YES')
920   FORMAT('OSOUNDING CURRENT SPEED (',A6,'):'//6X,9F10.2)
930   FORMAT('OSOUNDING WATER DEPTHS (',A6,'):'//6X,9F10.2)
      END
```

CHAPTER IV USING THE COMPUTER PROGRAM

4.1 Preparation of the Water Depth Grid. Once a coastal area is selected for making wave forecasts a water depth grid must be prepared. Details with numerous illustrations for preparing water depth grids are given by Wilson (1966). It is necessary to obtain charts of the region of interest containing sufficiently detailed bathymetric information.

A water depth grid is rectangular in shape. The value of x varies between 0 and AMM while y varies from 0 to ANN. The values of AMM and ANN are defined by

$$AMM = MM - 1 \tag{4-1}$$

$$ANN = NN - 1 \tag{4-2}$$

where MM is the number of water depth values in a y-column and NN is the number of columns. The value of MM must be an integral multiple of 16. If another number is preferred the format statement in the computer program used to input the water depth values must be changed. The maximum values of MM and NN depend upon the storage capacity of the computer. In the computer program presented in this report the values of MM and NN are assumed not to exceed 120. If the grid requirements exceed the storage capacity of the computer the coastal region of interest can be divided into several overlapping grids.

The xy-coordinate system is right-handed with the x-axis usually extending seaward. The direction of the x-axis with respect to true north is defined as CNVRSA. The use of CNVSRA makes it possible to define the input and output wave directions with respect to true north.

The distance between water depths in the x- or y-directions is a grid interval or grid unit and is denoted by GRID. This distance must be small enough for the water depth grid to describe adequately the bottom topography. If it is desirable for rays to start in deep water the grid must extend at least several grid units seaward of the deep water depth of the largest wave period of interest. In this report deep water is defined as any depth greater than 0.64 λ_d where λ_d is the deep water wavelength. This definition of deep water is chosen since the group speed is nearly invariant for greater water depths in the absence of currents.

To determine the location of the water depths to be read from a chart lines can be drawn on tracing paper parallel to the x- and y-axes of the grid and separated a distance equal to a grid unit. The tracing paper is placed on the chart and water depths are estimated for the points defined by the intersection of the grid lines. The water depths can be recorded in any system of units.

One of the program options is to have the shoreline drawn on a plot.

In order for the location of the shoreline to be computed it is necessary to determine negative values of water depths for at least two grid points landward of the shoreline. The negative values are determined by drawing the reflection of water depth contours on land with respect to the shoreline. Zero water depths can be used to fill out a column for grid points more than two grid units landward of the shore.

4.2 Preparation of the Current Grid. The current grid is prepared in a similar fashion to the water depth grid. The current grid contains both the x- and y-components of the current. The x-component current values follow the y-component values. The component values of the current can be either positive or negative, depending on the nature of the current. Both the x- and y-component values in the grid must have identical values of MM and NN. Any system of units can be used to record the current. As in the case of water depths, the grid unit spacing is denoted by GRID. If there is both a water depth grid and a current grid, the values of MM, NN, and GRID must be the same for both grids. It is not possible to determine a shoreline for a current.

4.3 Preparing a Computer Run. The way in which data is prepared for a computer run is illustrated on the coding form in Figure (4-1). Eight types of computer cards are used. The columns available for each parameter are outlined by rectangles. The positions of decimal points for real numbers are indicated. If there is no decimal point the number is an integer and is placed in the rectangle as far to the right as possible. The input parameters must appear on each card as shown, and the card types must be in the order indicated.

For the first type of computer card, MXPLOT is the number of runs for a given operation of the computer program. The PROJCT is a 6-character label of any combination of letters and numbers. The label can be used, for example, to indicate a project number. An alternative use is to identify which water depth grid is used for the run. It appears in both the printed output and on the plot. DATE1 and DATE2 are used to date the run. DATE1 can be used for the year and the month in the form ZZ/YY/. DATE2 can be used for the day in the form XX. The DIR is another 6-character label of any combination of letters and numbers. This label appears only on the plot. One possible label is WAVPAK, which can denote that wave packet trajectories are presented. If the rays have a common initial direction, DIR can be used to indicate that direction.

The number of rays for a given run, NOR, is input on the second type of computer card. The values of NPT and NSK determine the amount of printed output. If NPT is zero there is only one line of printout of the ray particulars for a given ray point. If NPT is not zero there are two lines of printout of the ray particulars (described in the next section). Printout occurs for the first ray point, those points which are an integral multiple of NSK, and the last point. The value of HT is the length of the y-axis of the plot in inches or centimeters. If CIN is not zero tick marks are placed on the rays at equal intervals of travel time given by the value of CIN in seconds. If no tick marks are desired CIN is zero.

The x- and y-axes of the plot will be calibrated and labeled if NAX is not zero. If NAX is zero the plot borders are drawn but the axes are not calibrated. The shoreline is drawn on the plot if NSH is not zero. If the shoreline is not desired NSH must be zero. In order to have a shoreline

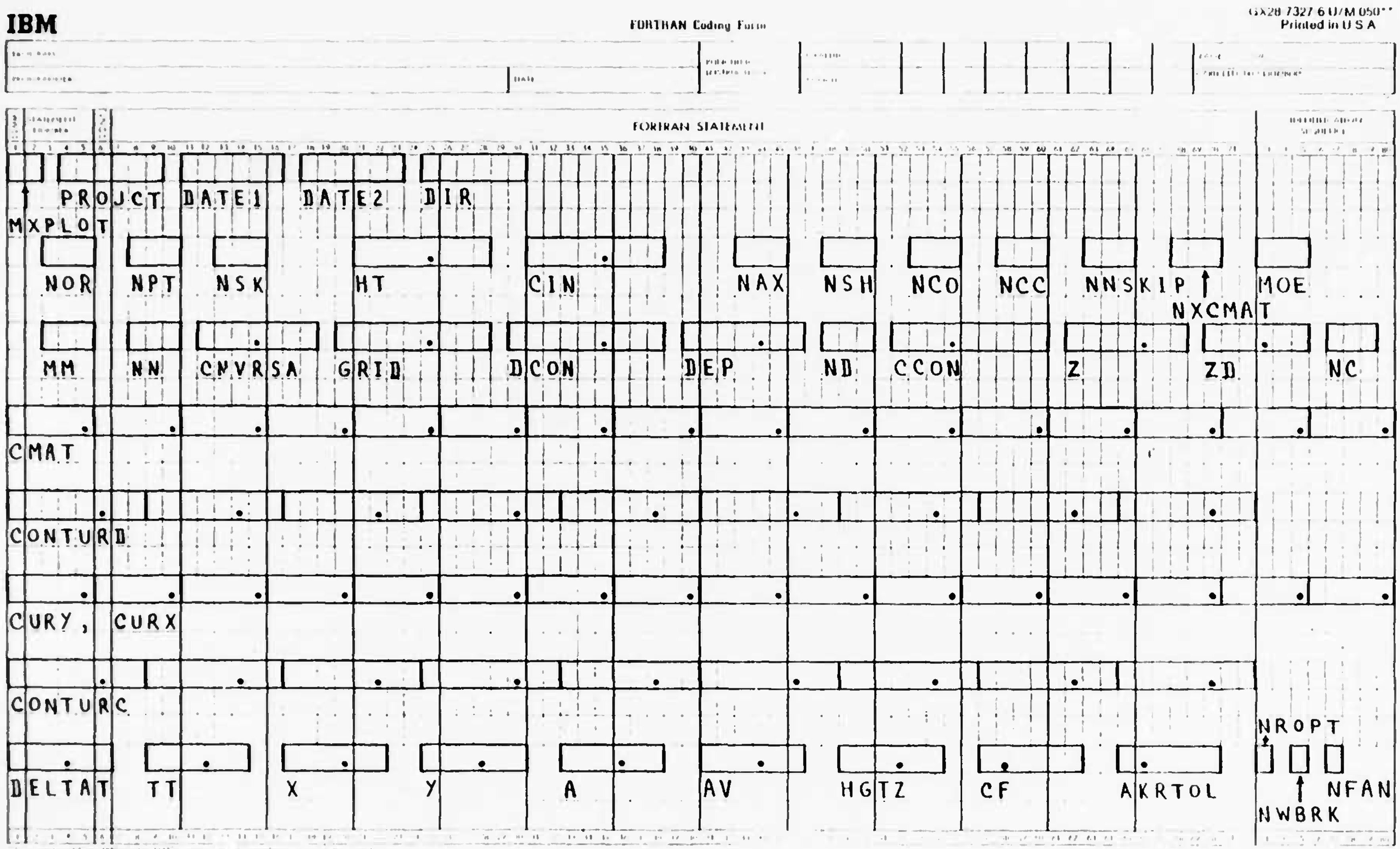


Figure (4-1). FORMAT OF THE INPUT PARAMETERS

there must be a water depth grid (CMAT). It must be prepared for a shoreline as described in Section (4.1). The number of sounding water depths for a plot is NCO. There cannot be more than 9 sounding depths, and they are input on the CONTURD computer card. If NCO is zero there are no sounding depths; in this case the CONTURD card must be removed from the input. In the same manner, NCC is the number of current speed contours for a plot. The contour values are input on the CONTURC card. Up to nine values can be placed on the card. If no current speed contours are desired the CONTURC card must not appear in the input and NCC must equal zero. The value of NNSKIP is the amount by which y is incremented in selecting columns for locating water depth and current speed contours. For example, if NNSKIP is 15 and NN is 64, sounding water depths are located for the 2, 17, 32, 47, and 62 y-columns. Current speed contours are located for the 4, 19, 34, and 51 y-columns.

If a water depth grid, a current grid, or both are to be read in the input for the computer run the value of NXCMAT is zero. If NXCMAT is not zero the grids for the previous run are used again. This situation can arise if MXPLOT is greater than one. If English units are to be used in the input and output MOE is zero. If MOE is not zero Metric units are used.

The third type of computer card contains the input dimensions for the water depth grid. A description of the quantities MM, NN, CNVRSA, and GRID are described in the previous sections of this chapter. The angle CNVRSA is given in degrees and GRID is given in feet or meters. The value of DCON is chosen so that the product of DCON and a water depth in CMAT yields a value with units of feet or meters.

A water depth value in feet or meters is read in the input as DEP. This value determines the water depth for a current grid if there is no water depth grid. If there is a water depth grid the value of DEP is replaced by values determined from the water depth grid. If a water depth grid is to be read in the input ND must equal one. Otherwise ND equals zero.

The value of CCON is chosen so that the product of CCON and a component current speed value in the current grid produces a value with units of feet/second or meters/second. A current speed magnitude can be input as Z and the current direction as ZD. These values represent a constant current to be used with a water depth grid provided there is no current grid. The values of Z and ZD are replaced with values determined from a current grid if one exists. If a current grid is to be read in the input NC must equal one. If NC equals zero no current grid is read.

The fourth type of computer card is used to input the water depth grid (CMAT). The units of CMAT determine the value of DCON. There are 16 water depths on each card. The water depths are entered column by column starting with the first column. There are NN columns. In each column the water depths are entered starting with the land values, if any, and proceeding seaward. There are MM values per column. The format for entering the water depths does not include numbers beyond the decimal points. Near shore it may be desirable to record water depths to the nearest tenth of a foot or meter. On some computer systems it is possible to enter data routinely in this form with the indicated format for CMAT being overridden. If such a capability is not available it may be desirable to alter the format statement for CMAT in WAVPAK.

If NCO is not zero the CONTURD computer card is used to input the sounding water depths in feet or meters. The number of sounding depths must agree with NCO which should not exceed 9. If NCO is zero the CONTURD card must be removed.

The current grid contains both CURX and CURY, and it appears as input on the sixth type of computer card. The CURY values precede the CURX values. The numbers are placed on the cards in the same manner as described for the CMAT cards.

The seventh type of computer card is CONTURC, and it contains the current speed contour values in feet/second or meters/second. The number of contour values must be equivalent to NCC. There cannot be more than 9 values. The CONTURC card must not appear in the input if NCC is zero.

The eighth type of computer card is used to input the particulars for each ray. There must be as many ray cards as declared in the input for NOR. The initial time step interval between ray points in seconds is DELTAT. The initial wave period in seconds is TT, and X, Y are the initial ray coordinates. The initial ray points should be at least two grid units from a grid boundary. The initial wave packet and wavelet directions are A and AV, respectively. The directions are in degrees and are the directions from which the waves come with respect to true north. The initial wave height in feet or meters is HGTZ.

The friction factor is CF. The value of AKRTOL determines the accuracy of the calculations of the refraction coefficient with the exception of near reflection points. As a general rule, if accuracy is required to the second decimal point AKRTOL is 0.01. If accuracy is desired to the third decimal point AKRTOL is 0.001.

To continue a ray beyond a reflection point NROPT is set unequal to zero. If NROPT is zero a ray is stopped at a reflection point. A test is made to determine if a wave breaks if NWBRK is not zero. If NWBRK is zero there is no test to determine if a wave breaks. If the ray is to be numbered at its terminal point NFAN is set unequal to zero. A group of rays should be numbered at their terminal points if they have a common origin. If NFAN is zero the ray is numbered at its initial point.

A sample of input data for a computer run with 6 rays is shown in Figure (4-2). Since the water depth contours are parallel, only one of the 64 columns of water depth values is shown in the rectangle labeled CMAT. The current speed contours also are parallel. The rectangle labeled CURY shows the current values for a column. The CURX values, which also must appear in the input, are all zero. Computer outputs for these data are presented in Section (4.6). Therefore, if desired, these input data can be used to check the computer program.

4.4 The Printed Output. At the beginning of each printout is a summary (Figure (4-4)) of the input information common to all the rays. The next thing that appears in the printout is the page heading. This contains the PROJCT, date, plot number, input wave period, ray number, input time step, friction factor, and AKRTOL. If at the first ray point this is followed by a statement denoting whether English or Metric units are used in the output. The column headings appear next in the output. Beyond the first point of a ray the page and column headings occur after every 50 lines of additional printout.

The column headings identify the ray particulars which appear in the output. They contain the ray point number MAX, the ray coordinates X, Y, and the water depth DEPTH in meters or feet. The current speed CUR:SP is in meters/second or feet/second, and the current direction CUR:DI is in degrees. The Doppler shifted wave period appears in the output as PERIOD. The ray, wave packet,

IBM FORTRAN Coding Form

GX28-7327-6 U/M 050** Printed in U.S.A.

FORTRAN STATEMENT

```
 1 PAR 2  82/09/ 15     WAVPAK
    6    1   10       15.24    300.0        1    1    4    0   15    0    1
   64   64180.0      750.0       1.0    500.0    1   0.01      0.0  270.0  000
 100.0    200.0   300.0 400.0
 25.0    20.0     4.0    60.0    15.0    15.0     3.0   0.1     0.001   1 1 1
 25.0    20.0     4.0    60.0    23.0    23.0     3.0   0.1     0.001   1 1 1
 25.0    12.0    25.0    60.0   165.0   165.0     3.0   0.1     0.001   0 1 1
 25.0    12.0    25.0    60.0   120.0   120.0     3.0   0.1     0.001   0 1 1
 25.0    17.0    42.0     3.0   200.0   200.0     3.0   0.0     0.001   0 0 1
 25.0    17.0    42.0     3.0   220.0   220.0     3.0   0.0     0.001   0 0 1

-15.0 -7.5  0.0  7.5 15.0 22.5 30.0 37.5 45.0 52.5 60.0 67.5 75.0 82.5 90.0 97.5
105.0112.5120.0127.5135.0142.5150.0157.5165.0172.5180.0187.5195.0202.5210.0217.5
225.0232.5240.0247.5255.0262.5270.0277.5285.0292.5300.0307.5315.0322.5330.0337.5
345.0352.5360.0367.5375.0382.5390.0397.5405.0412.5420.0427.5435.0442.5450.0457.5
CMAT

  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 19.5 38.0 55.5 72.0 87.5
102.0115.5128.0139.5150.0159.5168.0175.5182.0187.5192.0195.5198.0199.5200.0199.5
198.0195.5192.0187.5182.0175.5168.0159.5150.0139.5128.0115.5102.0 87.5 72.0 55.5
 38.0 19.5  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
CURY
```

Figure (4-2). SAMPLE INPUT DATA FOR A COMPUTER RUN

and wavelet directions are denoted, respectively, by RAY, PACK, and WAVE. These are the directions in degrees from which the waves come with respect to true north. The geometric group speed is given by G, and the ray speed is specified by GR. The units are in meters/second or feet/second. The wave height is HGT in meters or feet. The shoaling coefficient is identified by KS, and KF represents the friction coefficient. The refraction coefficient is defined by KR.

There is an additional line of printout if NPT is unequal to zero. This line contains fourteen ray particulars. The angle in degrees by which the x'y'-coordinate system (water depths) is rotated with respect to the positive x-axis for computations is given by ROTAT:D. An estimate of how well the computed water bottom surface fits the actual water depths is given by PCT:D (see page 59). The smaller the value the better the fit. The angle in degrees by which the x"y"-coordinate system (currents) is rotated with respect to the positive x-axis for computations is denoted by ROTAT:C. The quantities PCT:CX and PCT:CY are estimates of how well the surface fit derived x- and y-component current values, respectively, agree with the actual values (see page 59). The speed of the advected group front is GT, the conventional group speed is U, the phase speed is V, and the speed of the advected wavelet front is VT. These speeds have units of meters/second or feet/second. The ray separation factor is specified by BETA, and the time derivative of the ray separation factor is given by DBETA/DT. The type of time step breakup, should one occur, is shown as BRK UP. If the breakup is to maintain accuracy in the refraction calculations, BETA appears in the output. If the breakup is to keep the change in PACK to less than one degree between successive ray points near a reflection point, REFLECT appears in the printout. The number of intervals the input time step is divided into is denoted by NO. Finally, the wave packet ray curvature is given as CURVATURE in radians per grid unit.

The travel time along the wave packet trajectory does not appear in the output. However, it can be determined by subtracting one from MAX and multiplying the result by the input time step.

The ray particulars are printed out for the first ray point, those points which are an integral multiple of SK, and the last point. Printout occurs for a reflection point should one occur. Note that the number of ray points, NO, which occur when there is a breakup of the input time step interval is not counted in MAX. There is no printout for ray points which occur within a breakup.

A number of descriptive printouts appear with the ray particulars when certain types of calculations occur or when a ray terminates. If the packet ray curvature has not converged after fifty iterations the ray curvatures of the 48th and 50th iterations are compared to see if they are nearly the same. If so, the ray curvature is assumed to have converged to two values. This would happen if estimates of the new ray point alternate between two grid cells. For this situation, the average of the ray curvature averages for the 49th and 50th iterations is determined and is used to locate the next ray point. When this happens the following type of printout appears.

(1) MAX= 123, PACKET CURVATURE AVERAGED

If NSK > 1 this descriptive printout occurs even if there is no printed output of the ray particulars.

When there is a reflection one of three types of descriptive printouts occurs (Sections (2.1), f and g).

(2) MAX= 123, REFLECTION: SNELLS LAW WITH PHASE VELOCITY

(3) MAX= 123, REFLECTION: PACKET CURVATURE ITERATION NOT CONVERGING

(4) MAX= 123, REFLECTION: NEAR REFLECTION POINT

The ray point where one of these three descriptive printouts occurs is the last ray point if NROPT = 0.

When a ray terminates one of the following descriptive printouts can appear in the output.

(5) DIMENSION OF OUTPUT-ARRAYS EXCEEDED

(6) RAY REACHED SHORE

(7) RAY REACHED GRID BOUNDARY

(8) PACKET CURVATURE ITERATION NOT CONVERGING

(9) CAUSTIC OR FOCAL POINT

(10) WAVE BREAKS

(11) REFLECTION HAND-UP

(12) BREAKUP TIME STEP LESS THAN 0.5 SECOND

Printout (5) occurs if the sum of the number of ray points and tick marks is equal to or greater than the array dimension MMAX. Printout (6) is obtained if the water depth becomes zero or negative. Printout (7) results if the ray point is within 1.5 grid units of a grid boundary. The conditions for a reflection point are not satisfied if Printout (8) occurs. Printout (9) is produced if the ray separation factor becomes less than 0.0001. The condition for Printout (10) is given in Section (2.5). Printout (11) is obtained if there are successive reflections at the same ray point. Printout (12) can occur if the calculation time step becomes too small in either calculating the ray path near a reflection point (Section (2.1), f) or in calculating the ray separation factor (Section (2.3), c).

4.5 The Plots. Each plot contains a label consisting of PROJCT, the date, the scale factor, the time in seconds between tick marks on a ray, if any, the plot number, and DIR. If NAX $\neq$ 0 the axes of the plot are calibrated and labeled. If NSH $\neq$ 0 the shoreline is drawn. Each ray is numbered. If FAN = 0 the number appears at the initial ray point, and if FAN $\neq$ 0 the ray is numbered at its terminal point. If NCO $\neq$ 0 sounding water depth values are labeled. A "D" appears after the number to indicate a water depth. If NCC $\neq$ 0 current contour values are labeled. The number is followed by a "C" to signify a current value.

4.6 Examples of Computer Output. Several computer runs were made using the data presented in Figure (4-2). The six rays shown in the figure were

determined for the given water depth grid (CMAT). Each column of CMAT is the same as shown in Figure (4-2). A plot of the rays appears in Figure (4-3). Figure (4-4) shows a summary of the input parameters which are common to all the rays for this run. Figures (4-5) through (4-10) contain the printed output for the six rays. The examples illustrate rays beginning both at an intermediate water depth and in deep water. Three different wave periods are considered. The second ray undergoes a reflection (Section (2.1), f and Section (2.3), d). For the last two rays the friction factor is assumed to be zero. Tick marks and sounding water depths are shown on the plot.

The first and last rays for the data shown in Figure (4-2) were computed for the given current grid. Only the values for a column of CURY are shown. The CURX values are all zero. Figure (4-11) is a plot of the rays, and the printed output is presented in Figures (4-12) and (4-13).

For an additional comparison, the first and last rays in Figure (4-2) were computed for the given water depth and current grids. The resulting plot is seen in figure (4-14), and the listing of the rays is given in Figures (4-15) and (4-16).

In Figure (4-17) the period of the wave packet trajectories is 14.0 seconds and they begin in deep water. The water depth contours are sinusoidal with an amplitude of 5 kilometers and a wavelength of 20 kilometers. GRID has a value of 312.5 meters.

This example is quite interesting since there is decidedly more energy in the bay than at the headland. The opposite result would be expected for monochromatic trajectories. The refraction of wave packets could explain why there is more erosion in bays than at headlands for some coastlines. Figures (4-18) through (4-20) show the printed output for rays number 4, 8, and 16, respectively, of Figure (4-17). The computed refraction coefficients agree favorably with values estimated from the plot.

Figure (4-21) shows a portion of the Gulf of Mexico off the southwestern Florida coast. A water depth grid was prepared for this region with GRID = 14886.2 feet (4.537 km) and CNVRSA = 180°. A ray plot for this region is shown in Figure (4-22). The first two rays start at an intermediate water depth, whereas the remaining rays begin in deep water. Figure (4-23) displays printed output for the first portion of ray number 1. Since the water depth contours are not parallel there is a variation in ROTAT:D.

Ray number 2 has a reflection. Figure (4-24) shows a listing of the ray particulars near the reflection point. The wave packet and wavelet angles in the xy- and x'y'-coordinate systems are defined by Equations (3-1), (3-2), (2-111), and (2-112). At the reflection point the angles in the xy-coordinate system are $\theta_C = 275.22^{\circ}$ and $\gamma_C = 2.50^{\circ}$. In the x'y'-coordinate system $\theta' = 2.21^{\circ}$ and $\gamma' = 89.49^{\circ}$.

Ray number 12 illustrates the importance of examining the ray particulars in the printout. Figure (4-26) shows the printed output for this ray. A message in the output states there is a reflection. However, a reflection is not likely since DEPTH, θ', γ', CURVATURE, and G do not exhibit the behavior characteristic of a reflection. At the ray point where the reflection is indicated $\theta' = 115.33^{\circ}$ and $\gamma' = 117.07^{\circ}$. This false reflection is the result of a large change in ROTAT:D between successive ray points. When this occurs the water depth grid is not sufficiently detailed to adequately represent the changing water depth contours.

Figure (4-25) is the printout for ray number 6 of Figure (4-22). This is an example of a ray which reached shore.

PROJ. NO. PAR 2 , 82/09/15
SCL = 1/310039 , CIN = 300
PLOT NO. 1 , DIR. = WAVPAK

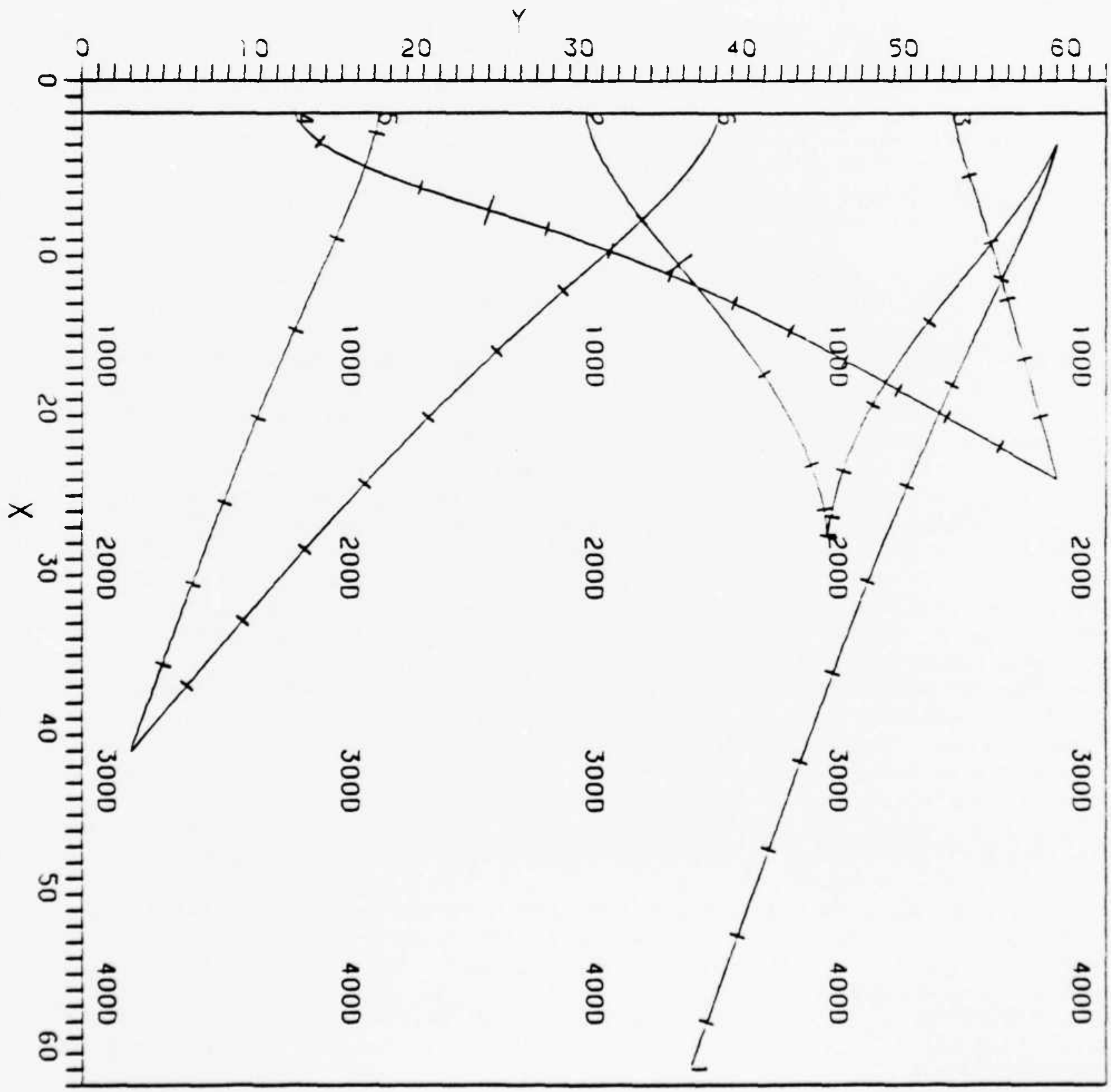


Figure (4-3). PLOT FOR SAMPLE INPUT DATA FOR WATER DEPTH GRID

```
WAVPAK -- WAVE PACKET TRAJECTORY ANALYSIS PROGRAM V2.0

SYSTEM PARAMTERS FOR PLOT NUMBER  1:

      PROJECT NAME ................................. PAR 2
      PROJECT DATE ................................. 82/09/15
      PLOT IDENTIFICATION LABEL .................... WAVPAK
      NUMBER OF RUNS TO BE MADE ....................   1

      DATA UNITS ................................... METRIC
      NUMBER OF RAYS TO BE RUN .....................   6
      PRINT INTERMEDIATE RAY RESULTS ............... YES
      INTERMEDIATE RAY RESULT INTERVAL ..........    10
      RAY TICK MARK SPACING ........................  300.000 SEC
      PLOTTER Y-AXIS LENGTH ........................    6.000 IN
      CALIBRATE AND LABEL AXES ..................... YES
      PLOT SHORELINE ............................... YES
      NUMBER OF SOUNDING DEPTHS ....................   4
      READ A NEW GRID .............................. YES
      GRID DIMENSIONS .............................. ( 64, 64)
      GRID SPACING .................................  2.286E+02 METERS
      RAY CONVERSION ANGLE .........................  180.000
      WATER DEPTH CONVERSION FACTOR.................  3.048E-01
      CURRENT SPEED CONVERSION FACTOR ..............  1.000E-02
      SOUNDING  INCREMENT ..........................  15
      WATER DEPTH GRID SPECIFIED ................... YES
      CURRENT GRID SPECIFIED ....................... NO
         INITIAL CURRENT SPEED .....................  0.000E+00
         INITIAL CURRENT DIRECTION .................  2.700E+02

SOUNDING WATER DEPTHS (METRIC):

         100.00     200.00     300.00     400.00
```

Figure (4-4). SUMMARY OF INPUT PARAMETERS COMMON TO ALL RAYS

PROJECT NO. PAR 2 , 82/09/15 , PLOT NO. 1, PERIOD= 20.0SEC., RAY NO. 1, DELTAT= 25.00, CF=0.100000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS. DEPTH,HGT(METER). G,GR,GT,U,V,VT(METER/SECOND).

MAX	X	Y	DEPTH	CUR:SP	CUR:DI	PERIOD	RAY	PACK	WAVE	G	GR	HGT	KS	KF	KR
	ROTAT:D	PCT:D	ROTAT:C	PCT:CX	PCT:CY	GT	U	V	VT	BETA	DBETA/DT	BRK UP	NO	CURVATURE	
1	4.00	60.00	15.00	0.0000	270.00	20.0000	15.00	15.00	15.00	11.24	11.24	3.0000	1.0000	1.0000	1.0000
	0.00	0.01	0.00	0.00	0.00	11.24	11.24	11.82	11.82	1.0000	-2.198E-04		1	-5.478E-02	
10	8.10	58.54	45.73	0.0000	270.00	20.0000	22.67	22.67	25.33	16.72	16.72	2.2645	0.8197	0.9003	1.0228
	0.00	0.01	0.00	0.00	0.00	16.72	16.74	19.54	19.54	0.9559	-1.452E-04	BETA	2	-1.631E-02	
20	13.55	56.07	86.63	0.0000	270.00	20.0000	25.20	25.20	33.03	18.47	18.47	2.1416	0.7800	0.8860	1.0328
	-0.01	0.01	0.00	0.00	0.00	18.47	18.64	24.89	24.89	0.9374	-1.191E-05		1	-1.060E-03	
30	19.08	53.50	128.08	0.0000	270.00	20.0000	24.49	24.49	37.65	17.97	17.97	2.1559	0.7907	0.8825	1.0299
	-0.01	0.01	0.00	0.00	0.00	17.97	18.46	27.89	27.89	0.9428	4.204E-05		1	4.101E-03	
40	24.40	51.15	168.02	0.0000	270.00	20.0000	22.95	22.95	40.23	16.90	16.90	2.2073	0.8154	0.8814	1.0238
	-0.02	0.01	0.00	0.00	0.00	16.90	17.70	29.50	29.50	0.9540	4.292E-05		1	4.727E-03	
50	29.47	49.08	205.98	0.0000	270.00	20.0000	21.59	21.59	41.61	15.95	15.95	2.2599	0.8392	0.8810	1.0189
	-0.02	0.01	0.00	0.00	0.00	15.95	16.98	30.33	30.33	0.9632	3.033E-05		1	3.748E-03	
60	34.32	47.21	242.37	0.0000	270.00	20.0000	20.63	20.63	42.33	15.29	15.29	2.3009	0.8572	0.8809	1.0157
	-0.02	0.01	0.00	0.00	0.00	15.29	16.46	30.76	30.76	0.9693	1.903E-05		1	2.561E-03	
70	39.03	45.47	277.69	0.0000	270.00	20.0000	20.02	20.02	42.70	14.86	14.86	2.3290	0.8694	0.8808	1.0137
	-0.03	0.01	0.00	0.00	0.00	14.86	16.11	30.97	30.97	0.9731	1.137E-05		1	1.619E-03	
80	43.65	43.81	312.33	0.0000	270.00	20.0000	19.65	19.65	42.88	14.61	14.61	2.3467	0.8770	0.8808	1.0126
	-0.03	0.01	0.00	0.00	0.00	14.61	15.90	31.08	31.08	0.9753	6.630E-06		1	9.771E-04	
90	48.21	42.19	346.57	0.0000	270.00	20.0000	19.43	19.43	42.98	14.46	14.46	2.3575	0.8816	0.8808	1.0119
	-0.03	0.01	0.00	0.00	0.00	14.46	15.77	31.14	31.14	0.9765	3.806E-06		1	5.725E-04	
100	52.74	40.60	380.55	0.0000	270.00	20.0000	19.30	19.30	43.03	14.37	14.37	2.3637	0.8843	0.8808	1.0116
	-0.04	0.01	0.00	0.00	0.00	14.37	15.69	31.17	31.17	0.9773	2.160E-06		1	3.286E-04	
110	57.25	39.02	414.34	0.0000	270.00	20.0000	19.25	19.25	43.08	14.27	14.27	2.3717	0.8874	0.8808	1.0114
	-0.04	0.01	0.00	0.00	0.00	14.27	15.60	31.19	31.19	0.9775	1.621E-06		1	0.000E+00	
120	61.74	37.45	448.01	0.0000	270.00	20.0000	19.25	19.25	43.08	14.27	14.27	2.3717	0.8874	0.8808	1.0114
	-0.04	0.01	0.00	0.00	0.00	14.27	15.60	31.19	31.19	0.9775	1.621E-06		1	0.000E+00	

RAY REACHED GRID BOUNDARY

Figure (4-5). LISTING FOR RAY NUMBER 1 OF SAMPLE INPUT DATA FOR WATER DEPTH GRID

PROJECT NO. PAR 2 , 82/09/15 , PLOT NO. 1, PERIOD= 20.0SEC., RAY NO. 2, DELTAT= 25.00, CF=0.100000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS. DEPTH,HGT(METER). G,GR,GT,U,V,VT(METER/SECOND).

MAX	X	Y	DEPTH	CUR:SP	CUR:DI	PERIOD	RAY	PACK	WAVE	G	GR	HGT	KS	KF	KR
	ROTAT:D	FCT:D	ROTAT:C	FCT:CX	FCT:CY	GT	U	V	VT	BETA	DBETA/DT	BRK UP	NO	CURVATURE	
1	4.00	60.00	15.00	0.0000	270.00	20.0000	23.00	23.00	23.00	11.24	11.24	3.0000	1.0000	1.0000	1.0000
	0.00	0.01	0.00	0.00	0.00	11.24	11.24	11.82	11.82	1.0000	-5.258E-04		1	-8.270E-02	
10	7.71	57.86	42.82	0.0000	270.00	20.0000	34.80	34.80	38.93	16.40	16.40	2.3481	0.8277	0.8938	1.0581
	0.00	0.01	0.00	0.00	0.00	16.40	16.44	19.01	19.01	0.8932	-3.608E-04	BETA	2	-2.660E-02	
20	12.34	54.31	77.55	0.0000	270.00	20.0000	38.64	38.64	52.41	17.95	17.95	2.2551	0.7912	0.8758	1.0848
	-0.01	0.01	0.00	0.00	0.00	17.95	18.48	23.97	23.97	0.8498	1.514E-05	BETA	2	9.041E-04	
30	16.94	50.80	112.05	0.0000	270.00	20.0000	34.99	34.99	62.93	16.48	16.48	2.2835	0.8257	0.8706	1.0589
	-0.01	0.01	0.00	0.00	0.00	16.48	18.65	26.94	26.94	0.8919	2.866E-04		1	2.073E-02	
40	21.12	48.25	143.42	0.0000	270.00	20.0000	27.04	27.04	71.13	13.07	13.07	2.4518	0.9273	0.8685	1.0148
	-0.01	0.01	0.00	0.00	0.00	13.07	18.19	28.63	28.63	0.9711	3.069E-04		1	3.534E-02	
50	24.47	46.83	168.51	0.0000	270.00	20.0000	18.70	18.70	77.32	9.21	9.21	2.8263	1.1043	0.8675	0.9834
	-0.02	0.01	0.00	0.00	0.00	9.21	17.69	29.51	29.51	1.0340	1.915E-04		1	4.472E-02	
60	26.82	46.18	186.11	0.0000	270.00	20.0000	11.78	11.78	82.02	5.86	5.86	3.5051	1.3842	0.8668	0.9738
	-0.02	0.01	0.00	0.00	0.00	5.86	17.34	29.96	29.96	1.0545	1.384E-04		1	5.663E-02	
70	28.23	45.96	196.68	0.0000	270.00	20.0000	6.17	6.17	85.80	3.09	3.09	4.8280	1.9080	0.8661	0.9738
	-0.02	0.01	0.00	0.00	0.00	3.09	17.14	30.17	30.17	1.0545	1.384E-04		1	9.035E-02	
80	28.81	45.91	201.05	0.0000	270.00	20.0000	1.45	1.45	89.03	0.72	0.72	9.9758	3.9463	0.8653	0.9738
	-0.02	0.01	0.00	0.00	0.00	0.72	17.07	30.25	30.25	1.0545	1.384E-04		1	3.515E-01	
81	28.83	45.91	201.18	0.0000	270.00	20.0000	1.02	1.02	89.32	0.51	0.51	11.9000	4.7083	0.8651	0.9738
	-0.02	0.01	0.00	0.00	0.00	0.51	17.06	30.25	30.25	1.0545	1.384E-04		1	4.897E-01	
MAX = 81,		REFLECTION:	NEAR REFLECTION POINT												
90	28.24	45.87	196.80	0.0000	270.00	20.0000	173.82	173.82	94.19	3.08	3.08	4.8200	1.9086	0.8644	0.9738
	-0.02	0.01	0.00	0.00	0.00	3.08	17.14	30.18	30.18	1.0545	-1.384E-04		1	9.312E-02	
100	26.69	45.61	185.19	0.0000	270.00	20.0000	167.61	167.61	98.30	6.13	6.13	3.4161	1.3537	0.8638	0.9738
	-0.02	0.01	0.00	0.00	0.00	6.13	17.36	29.94	29.94	1.0545	-1.384E-04		1	5.649E-02	
110	24.12	44.85	165.87	0.0000	270.00	20.0000	160.03	160.03	103.38	9.75	9.75	2.7488	1.0732	0.8631	0.9892
	-0.02	0.01	0.00	0.00	0.00	9.75	17.74	29.44	29.44	1.0219	-2.309E-04		1	4.420E-02	
120	20.53	43.21	138.98	0.0000	270.00	20.0000	151.16	151.16	110.02	13.76	13.76	2.4003	0.9036	0.8619	1.0274
	-0.01	0.01	0.00	0.00	0.00	13.76	18.27	28.43	28.43	0.9474	-3.554E-04	BETA	2	3.368E-02	
130	16.18	40.37	106.32	0.0000	270.00	20.0000	143.44	143.44	118.72	16.98	16.98	2.2628	0.8134	0.8594	1.0789
	-0.01	0.01	0.00	0.00	0.00	16.98	18.69	26.54	26.54	0.8591	-3.066E-04		1	1.742E-02	
140	11.54	36.70	71.56	0.0000	270.00	20.0000	140.96	140.96	129.68	17.95	17.95	2.2421	0.7911	0.8530	1.1076
	-0.01	0.01	0.00	0.00	0.00	17.95	18.31	23.29	23.29	0.8152	-1.425E-05	BETA	2	-3.288E-03	
150	6.99	33.26	37.44	0.0000	270.00	20.0000	146.36	146.36	143.63	15.79	15.79	2.2682	0.8434	0.8306	1.0793
	0.00	0.01	0.00	0.00	0.00	15.79	15.81	17.95	17.95	0.8584	3.531E-04	BETA	2	-3.330E-02	
160	3.23	31.29	9.26	0.0000	270.00	20.0000	161.43	161.43	161.95	9.09	9.09	2.2922	1.1119	0.6769	1.0152
	0.00	0.01	0.00	0.00	0.00	9.09	9.09	9.38	9.38	0.9703	4.482E-04	BETA	4	-1.169E-01	
										RAY REACHED SHORE					
167	2.03	31.02	0.25	0.0000	270.00	20.0000	176.88	176.88	177.04	1.56	1.56	0.2098	2.6834	0.0263	0.9891
	0.00	999.00	0.00	0.00	0.00	1.56	1.56	1.56	1.56	1.0222	9.070E-05	BETA	4	-8.165E-01	

Figure (4-6). LISTING FOR RAY NUMBER 2 OF SAMPLE INPUT DATA FOR WATER DEPTH GRID

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PROJECT NO. PAR 2 ,  82/09/15    ,    PLOT NO.  1, PERIOD= 12.0SEC., RAY NO.  3, DELTAT= 25.00, CF=0.100000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS.  DEPTH,HGT(METER).  G,GR,GT,U,V,VT(METER/SECOND).

MAX  X        Y        DEPTH    CUR:SP   CUR:DI   PERIOD    RAY       PACK     WAVE     G         GR       HGT        KS          KF         KR
 ROTAT:D  PCT:D    ROTAT:C  PCT:CX   PCT:CY   GT       U         V        VT       BETA     DBETA/DT   BRK UP   NO      CURVATURE

  1   25.00    60.00    172.49   0.0000   270.00  12.0000   165.00   165.00   165.00    9.36      9.36     3.0000     1.0000     1.0000     1.0000
     -0.02     0.01      0.00     0.00      0.00    9.36     9.36     18.72    18.72   1.0000  0.000E+00               1   0.000E+00
 10   22.29    59.27    152.16   0.0000   270.00  12.0000   165.00   165.00   165.00    9.36      9.36     3.0000     1.0000     1.0000     1.0000
     -0.01     0.01      0.00     0.00      0.00    9.36     9.36     18.72    18.72   1.0000  0.000E+00               1   0.000E+00
 20   19.26    58.46    129.47   0.0000   270.00  12.0000   164.94   164.94   165.02    9.44      9.44     2.9869     0.9956     1.0000     1.0000
     -0.01     0.01      0.00     0.00      0.00    9.44     9.44     18.69    18.69   0.9999 -1.237E-06               1   7.935E-04
 30   16.21    57.63    106.55   0.0000   270.00  12.0000   164.71   164.71   165.08    9.59      9.59     2.9646     0.9878     0.9999     1.0004
     -0.01     0.01      0.00     0.00      0.00    9.59     9.59     18.62    18.62   0.9991 -5.750E-06               1   2.073E-03
 40   13.07    56.76     83.04   0.0000   270.00  12.0000   164.12   164.12   165.27    9.96      9.96     2.9116     0.9693     0.9996     1.0017
     -0.01     0.01      0.00     0.00      0.00    9.96     9.96     18.39    18.39   0.9966 -1.578E-05               1   4.620E-03
 50    9.78    55.79     58.33   0.0000   270.00  12.0000   162.95   162.95   165.92   10.68     10.68     2.8128     0.9362     0.9970     1.0045
     -0.01     0.01      0.00     0.00      0.00   10.68    10.69     17.59    17.59   0.9910 -2.741E-05               1   6.760E-03
 60    6.28    54.68     32.08   0.0000   270.00  12.0000   162.18   162.18   167.97   11.14     11.14     2.7068     0.9165     0.9783     1.0064
      0.00     0.01      0.00     0.00      0.00   11.14    11.20     15.08    15.08   0.9873  1.885E-05               1  -3.614E-03
 70    3.08    53.75      8.13   0.0000   270.00  12.0000   167.47   167.47   173.18    7.92      7.92     2.5113     1.0871     0.7750     0.9936
      0.00     0.01      0.00     0.00      0.00    7.92     7.96      8.59     8.59   1.0130  1.833E-04       BETA    2  -7.620E-02
                                                                                   RAY REACHED SHORE
 76    2.07    53.59      0.52   0.0000   270.00  12.0000   176.51   176.51   178.22    2.24      2.24     0.4574     2.0445     0.0759     0.9822
      0.00   999.00      0.00     0.00      0.00    2.24     2.24      2.25     2.25   1.0365  8.062E-05       BETA    4  -4.333E-01
```

Figure (4-7). LISTING FOR RAY NUMBER 3 OF SAMPLE INPUT DATA FOR WATER DEPTH GRID

PROJECT NO. PAR 2 , 82/09/15 , PLOT NO. 1, PERIOD= 12.0SEC., RAY NO. 4, DELTAT= 25.00, CF=0.100000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS. DEPTH,HGT(METER). G,GR,GT,U,V,VT(METER/SECOND).

MAX	X	Y	DEPTH	CUR:SP	CUR:DI	PERIOD	RAY	PACK	WAVE	G	GR	HGT	KS	KF	KR
	ROTAT:D	PCT:D	ROTAT:C	PCT:CX	PCT:CY	GT	U	V	VT	BETA	DBETA/DT	BRK UP	NO	CURVATURE	
1	25.00	60.00	172.49	0.0000	270.00	12.0000	120.00	120.00	120.00	9.36	9.36	3.0000	1.0000	1.0000	1.0000
	-0.02	0.01	0.00	0.00	0.00	9.36	9.36	18.72	18.72	1.0000	0.000E+00		1	0.000E+00	
10	23.60	57.57	161.96	0.0000	270.00	12.0000	120.00	120.00	120.00	9.36	9.36	3.0000	1.0000	1.0000	1.0000
	-0.02	0.01	0.00	0.00	0.00	9.36	9.36	18.72	18.72	1.0000	0.000E+00		1	0.000E+00	
20	22.04	54.87	150.27	0.0000	270.00	12.0000	120.00	120.00	120.00	9.36	9.36	3.0000	1.0000	1.0000	1.0000
	-0.01	0.01	0.00	0.00	0.00	9.36	9.36	18.72	18.72	1.0000	0.000E+00		1	0.000E+00	
30	20.47	52.16	138.55	0.0000	270.00	12.0000	119.88	119.88	120.08	9.41	9.41	2.9919	0.9971	1.0000	1.0002
	-0.01	0.01	0.00	0.00	0.00	9.41	9.41	18.70	18.70	0.9997	-7.194E-06		1	1.753E-03	
40	18.92	49.43	126.87	0.0000	270.00	12.0000	119.48	119.48	120.16	9.45	9.45	2.9928	0.9951	1.0000	1.0025
	-0.01	0.01	0.00	0.00	0.00	9.45	9.45	18.69	18.69	0.9949	-3.249E-05		1	2.904E-03	
50	17.38	46.67	115.32	0.0000	270.00	12.0000	118.82	118.82	120.31	9.51	9.51	3.0021	0.9918	0.9999	1.0090
	-0.01	0.01	0.00	0.00	0.00	9.51	9.52	18.66	18.66	0.9823	-7.114E-05		1	4.612E-03	
60	15.86	43.87	103.98	0.0000	270.00	12.0000	117.80	117.80	120.56	9.61	9.61	3.0245	0.9870	0.9998	1.0216
	-0.01	0.01	0.00	0.00	0.00	9.61	9.62	18.61	18.61	0.9581	-1.246E-04		1	6.874E-03	
70	14.40	41.00	92.97	0.0000	270.00	12.0000	116.32	116.32	121.01	9.74	9.74	3.0665	0.9804	0.9996	1.0430
	-0.01	0.01	0.00	0.00	0.00	9.74	9.77	18.52	18.52	0.9192	-1.870E-04		1	9.343E-03	
80	12.99	38.04	82.45	0.0000	270.00	12.0000	114.40	114.40	121.75	9.89	9.89	3.1332	0.9725	0.9990	1.0749
	-0.01	0.01	0.00	0.00	0.00	9.89	9.98	18.38	18.38	0.8654	-2.392E-04		1	1.119E-02	
90	11.68	34.99	72.58	0.0000	270.00	12.0000	112.22	112.22	122.85	10.06	10.06	3.2223	0.9646	0.9978	1.1161
	-0.01	0.01	0.00	0.00	0.00	10.06	10.23	18.16	18.16	0.8028	-2.530E-04		1	1.138E-02	
100	10.46	31.84	63.44	0.0000	270.00	12.0000	110.16	110.16	124.39	10.20	10.20	3.3157	0.9580	0.9951	1.1594
	-0.01	0.01	0.00	0.00	0.00	10.20	10.52	17.84	17.84	0.7439	-2.084E-04		1	9.429E-03	
110	9.33	28.62	54.95	0.0000	270.00	12.0000	108.66	108.66	126.40	10.29	10.29	3.3760	0.9535	0.9899	1.1922
	-0.01	0.01	0.00	0.00	0.00	10.29	10.81	17.40	17.40	0.7035	-1.050E-04		1	5.581E-03	
120	8.25	25.35	46.86	0.0000	270.00	12.0000	108.08	108.08	128.98	10.33	10.33	3.3553	0.9519	0.9802	1.1987
	0.00	0.01	0.00	0.00	0.00	10.33	11.05	16.80	16.80	0.6959	5.434E-05		1	9.545E-06	
130	7.17	22.09	38.75	0.0000	270.00	12.0000	108.83	108.83	132.32	10.28	10.28	3.2072	0.9540	0.9622	1.1646
	0.00	0.01	0.00	0.00	0.00	10.28	11.21	15.98	15.98	0.7372	2.945E-04		1	-8.27[illegible]E-03	
140	6.00	18.89	30.02	0.0000	270.00	12.0000	111.69	111.69	136.98	10.09	10.09	2.8953	0.9632	0.9281	1.0796
	0.00	0.01	0.00	0.00	0.00	10.09	11.16	14.75	14.75	0.8579	7.182E-04	BETA	2	-2.348E-02	
150	4.63	15.92	19.73	0.0000	270.00	12.0000	119.07	119.07	144.28	9.49	9.49	2.3865	0.9931	0.8541	0.9379
	0.00	0.01	0.00	0.00	0.00	9.49	10.49	12.62	12.62	1.1369	1.648E-03	BETA	2	-6.291E-02	
160	2.93	13.65	6.96	0.0000	270.00	12.0000	139.30	139.30	158.31	7.08	7.08	1.5868	1.1501	0.6138	0.7492
	0.00	999.00	0.00	0.00	0.00	7.08	7.48	7.99	7.99	1.7814	3.644E-03	BETA	8	-2.521E-01	
										RAY REACHED SHORE					
166	2.07	13.16	0.55	0.0000	270.00	12.0000	167.76	167.76	173.83	2.30	2.30	0.3445	2.0181	0.0863	0.6594
	0.00	999.00	0.00	0.00	0.00	2.30	2.31	2.32	2.32	2.3002	2.125E-03	BETA	16	-1.398E+00	

Figure (4-8). LISTING FOR RAY NUMBER 4 OF SAMPLE INPUT DATA FOR WATER DEPTH GRID

```
PROJECT NO. PAR 2 ,  82/09/15     ,     PLOT NO.  1, PERIOD= 17.0SEC., RAY NO.  5, DELTAT= 25.00, CF=0.000000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS.  DEPTH,HGT(METER).  G,GR,GT,U,V,VT(METER/SECOND).

MAX  X        Y        DEPTH    CUR:SF   CUR:DI   PERIOD    RAY      PACK     WAVE      G         GR     HGT        KS          KF        KR
 ROTAT:D  PCT:D    ROTAT:C  PCT:CX   PCT:CY   GT       U        V        VT       BETA      DBETA/DT   BRK UP    NO   CURVATURE

  1   42.00     3.00    299.99    0.0000   270.00  17.0000  200.00   200.00   200.00   13.26   13.26   3.0000    1.0000    1.0000    1.0000
    -0.03     0.01     0.00     0.00     0.00    13.26   13.26    26.52    26.52  1.0000  0.000E+00              1   0.000E+00
 10   38.25     4.37    271.88    0.0000   270.00  17.0000  200.04   200.04   199.98   13.35   13.35   2.9901    0.9967    1.0000    1.0000
    -0.03     0.01     0.00     0.00     0.00    13.35   13.35    26.49    26.49  1.0000 -7.740E-07              1  -3.987E-04
 20   34.06     5.90    240.45    0.0000   270.00  17.0000  200.18   200.18   199.95   13.44   13.44   2.9804    0.9932    1.0000    1.0003
    -0.02     0.01     0.00     0.00     0.00    13.44   13.44    26.45    26.45  0.9995 -3.374E-06              1  -7.923E-04
 30   29.84     7.46    208.76    0.0000   270.00  17.0000  200.46   200.46   199.88   13.62   13.62   2.9622    0.9865    1.0000    1.0010
    -0.02     0.01     0.00     0.00     0.00    13.62   13.62    26.36    26.36  0.9981 -8.249E-06              1  -1.499E-03
 40   25.54     9.08    176.54    0.0000   270.00  17.0000  200.97   200.97   199.72   13.96   13.96   2.9304    0.9744    1.0000    1.0025
    -0.02     0.01     0.00     0.00     0.00    13.96   13.97    26.16    26.16  0.9951 -1.659E-05              1  -2.615E-03
 50   21.12    10.81    143.43    0.0000   270.00  17.0000  201.85   201.85   199.34   14.53   14.53   2.8807    0.9552    1.0000    1.0053
    -0.01     0.01     0.00     0.00     0.00    14.53   14.54    25.68    25.68  0.9895 -2.803E-05              1  -3.921E-03
 60   16.54    12.71    109.03    0.0000   270.00  17.0000  203.05   203.05   198.49   15.29   15.29   2.8198    0.9312    1.0000    1.0094
    -0.01     0.01     0.00     0.00     0.00    15.29   15.34    24.59    24.59  0.9815 -3.318E-05              1  -4.166E-03
 70   11.77    14.79     73.24    0.0000   270.00  17.0000  203.83   203.83   196.67   15.78   15.78   2.7831    0.9166    1.0000    1.0121
    -0.01     0.01     0.00     0.00     0.00    15.78   15.90    22.24    22.24  0.9763  3.092E-06              1   9.816E-05
 80    7.07    16.80     38.01    0.0000   270.00  17.0000  201.87   201.87   193.12   14.55   14.55   2.8763    0.9545    1.0000    1.0045
     0.00     0.01     0.00     0.00     0.00    14.55   14.72    17.59    17.59  0.9910  1.306E-04              1   1.665E-02
 90    3.26    18.09      9.43    0.0000   270.00  17.0000  193.21   193.21   186.97    8.95    8.95   3.5802    1.2174    1.0000    0.9803
     0.00     0.01     0.00     0.00     0.00     8.95    9.00     9.40     9.40  1.0407  2.285E-04      BETA    2   7.792E-02
                                                                              RAY REACHED SHORE
 97    2.04    18.28      0.28    0.0000   270.00  17.0000  182.41   182.41   181.23    1.66    1.66   8.2026    2.8266    1.0000    0.9673
     0.00   999.00     0.00     0.00     0.00     1.66    1.66     1.66     1.66  1.0687  5.373E-05      BETA    4   5.573E-01
```

Figure (4-9). LISTING FOR RAY NUMBER 5 OF SAMPLE INPUT DATA FOR WATER DEPTH GRID

```
PROJECT NO. PAR 2 ,  82/09/15    ,    PLOT NO.  1, PERIOD= 17.0SEC., RAY NO.  6, DELTAT= 25.00, CF=0.000000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS.  DEPTH,HGT(METER).  G,GR,GT,U,V,VT(METER/SECOND).

MAX  X        Y        DEPTH    CUR:SF   CUR:DI    PERIOD    RAY      PACK     WAVE     G       GR      HGT         KS         KF          KR
 ROTAT:D  PCT:D    ROTAT:C  PCT:CX  PCT:CY  GT       U        V        VT       BETA    DBETA/DT  BRK UP   NO    CURVATURE

   1   42.00     3.00    299.99   0.0000   270.00  17.0000  220.00  220.00  220.00  13.26  13.26   3.0000   1.0000    1.0000   1.0000
    -0.03     0.01    0.00     0.00     0.00  13.26   13.26   26.52   26.52  1.0000  0.000E+00          1  0.000E+00
  10   38.95     5.56    277.08   0.0000   270.00  17.0000  220.06  220.06  219.96  13.34  13.34   2.9913   0.9971    1.0000   1.0000
    -0.03     0.01    0.00     0.00     0.00  13.34   13.34   26.49   26.49  0.9999 -2.050E-06          1 -6.658E-04
  20   35.54     8.44    251.55   0.0000   270.00  17.0000  220.28  220.28  219.91  13.40  13.40   2.9863   0.9947    1.0000   1.0007
    -0.02     0.01    0.00     0.00     0.00  13.40   13.40   26.47   26.47  0.9985 -9.637E-06          1 -1.169E-03
  30   32.13    11.35    225.97   0.0000   270.00  17.0000  220.67  220.67  219.83  13.51  13.51   2.9800   0.9907    1.0000   1.0027
    -0.02     0.01    0.00     0.00     0.00  13.51   13.51   26.42   26.42  0.9947 -2.232E-05          1 -1.986E-03
  40   28.71    14.32    200.32   0.0000   270.00  17.0000  221.32  221.32  219.66  13.69  13.69   2.9719   0.9841    1.0000   1.0067
    -0.02     0.01    0.00     0.00     0.00  13.69   13.70   26.32   26.32  0.9868 -4.227E-05          1 -3.213E-03
  50   25.27    17.39    174.56   0.0000   270.00  17.0000  222.35  222.35  219.32  13.98  13.98   2.9625   0.9740    1.0000   1.0139
    -0.02     0.01    0.00     0.00     0.00  13.98   13.99   26.14   26.14  0.9728 -7.034E-05          1 -4.809E-03
  60   21.83    20.61    148.68   0.0000   270.00  17.0000  223.85  223.85  218.68  14.38  14.38   2.9536   0.9602    1.0000   1.0253
    -0.02     0.01    0.00     0.00     0.00  14.38   14.44   25.78   25.78  0.9513 -1.014E-04          1 -6.346E-03
  70   18.37    24.04    122.76   0.0000   270.00  17.0000  225.71  225.71  217.52  14.86  14.86   2.9488   0.9446    1.0000   1.0406
    -0.01     0.01    0.00     0.00     0.00  14.86   15.01   25.12   25.12  0.9235 -1.155E-04          1 -6.715E-03
  80   14.91    27.70     96.84   0.0000   270.00  17.0000  227.38  227.38  215.53  15.27  15.27   2.9491   0.9317    1.0000   1.0551
    -0.01     0.01    0.00     0.00     0.00  15.27   15.61   23.97   23.97  0.8983 -7.246E-05          1 -4.173E-03
  90   11.47    31.48     71.01   0.0000   270.00  17.0000  227.64  227.64  212.29  15.34  15.34   2.9462   0.9297    1.0000   1.0563
    -0.01     0.01    0.00     0.00     0.00  15.34   15.90   22.03   22.03  0.8963  7.851E-05          1  3.457E-03
 100    8.02    35.11     45.19   0.0000   270.00  17.0000  224.51  224.51  207.16  14.55  14.55   2.9368   0.9545    1.0000   1.0256
     0.00     0.01    0.00     0.00     0.00  14.55   15.25   18.83   18.83  0.9507  3.874E-04          1  2.102E-02
 110    4.65    37.97     19.90   0.0000   270.00  17.0000  214.28  214.28  198.84  11.70  11.70   3.0401   1.0647    1.0000   0.9518
     0.00     0.01    0.00     0.00     0.00  11.70   12.13   13.32   13.32  1.1038  8.429E-04      BETA  2  7.151E-02
 120    2.20    39.15      1.51   0.0000   270.00  17.0000  190.52  190.52  185.34   3.80   3.80   4.8864   1.8686    1.0000   0.8717
     0.00   999.00    0.00     0.00     0.00   3.80    3.81    3.84    3.84  1.3161  5.403E-04      BETA  8  4.387E-01
                                                                               RAY REACHED SHORE
 122    2.03    39.18      0.24   0.0000   270.00  17.0000  184.16  184.16  182.11   1.52   1.52   7.6709   2.9549    1.0000   0.8653
     0.00   999.00    0.00     0.00     0.00   1.52    1.52    1.52    1.52  1.3355  2.219E-04      BETA  8  1.146E+00
```

Figure (4-10). LISTING FOR RAY NUMBER 6 OF SAMPLE INPUT DATA FOR WATER DEPTH GRID

PROJ. NO. PCURR , 82/09/15
SCL = 1/310039 , CIN = 0
PLOT NO. 1 , DIR. = WAVPAK

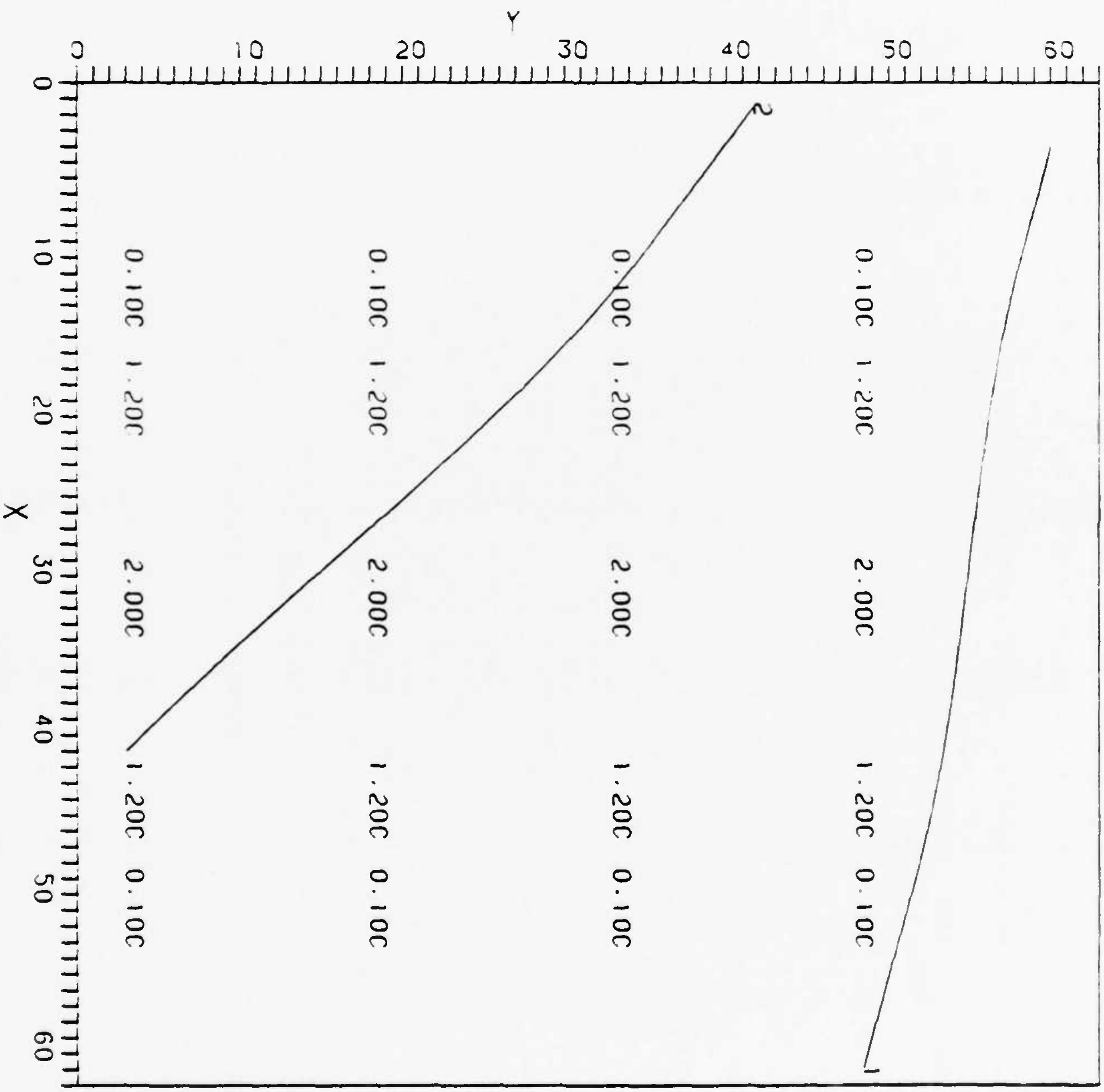

Figure (4-11). PLOT FOR SAMPLE INPUT DATA FOR CURRENT GRID

```
PROJECT NO. PCURR ,  82/09/15    ,     PLOT NO.  1, PERIOD= 20.0SEC., RAY NO.  1, DELTAT= 25.00, CF=0.100000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS.  DEPTH,HGT(METER).  G,GR,GT,U,V,VT(METER/SECOND).

MAX  X        Y        DEPTH    CUR:SF   CUR:DI    PERIOD    RAY     PACK     WAVE     G        GR      HGT        KS          KF          KR
 ROTAT:D  PCT:D   ROTAT:C  PCT:CX  PCT:CY  GT       U        V       VT       BETA    DBETA/DT   BRK UP   NO   CURVATURE

  1    4.00   60.00    500.00   0.0000   270.00  20.0000   15.00   15.00   15.00  15.60  15.60   3.0000   1.0000    1.0000    1.0000
     0.00    0.00      0.00  999.00  999.00   15.60   15.60   31.19   31.19  1.0000  0.000E+00           1  0.000E+00
 10    8.52   58.79    500.00   0.0000   270.00  20.0000   15.00   15.00   15.00  15.60  15.60   3.0000   1.0000    1.0000    1.0000
     0.00    0.00      0.00  999.00  999.00   15.60   15.60   31.19   31.19  1.0000  0.000E+00           1  0.000E+00
 20   13.53   57.53    500.00   0.6444   270.00  19.8936   12.77   14.77   14.84  15.51  15.36   3.0226   1.0080    1.0000    0.9995
     0.00    0.00      0.00  999.00    0.01   15.35   15.51   31.03   30.86  1.0010  5.402E-06           1  1.045E-03
 30   18.52   56.55    500.00   1.3414   270.00  19.7798    9.82   14.51   14.66  15.43  15.15   3.0469   1.0167    1.0000    0.9990
     0.00    0.00     -0.01  999.00    0.01   15.09   15.43   30.85   30.51  1.0021  3.512E-06           1  7.150E-04
 40   23.49   55.80    500.00   1.7883   270.00  19.7076    7.85   14.34   14.56  15.37  15.03   3.0625   1.0222    1.0000    0.9986
     0.00    0.00     -0.02  999.00    0.01   14.93   15.37   30.74   30.29  1.0028  1.889E-06           1  3.996E-04
 50   28.45   55.17    500.00   1.9880   270.00  19.6755    6.87   14.26   14.51  15.34  14.98   3.0694   1.0247    1.0000    0.9985
     0.00    0.00     -0.09  999.00    0.01   14.85   15.34   30.69   30.19  1.0031  4.026E-07           1  9.377E-05
 60   33.41   54.57    500.00   1.9417   270.00  19.6829    7.00   14.27   14.52  15.35  14.99   3.0678   1.0241    1.0000    0.9985
     0.00    0.00   -179.96  999.00    0.01   14.87   15.35   30.70   30.21  1.0030 -8.447E-07           1 -2.095E-04
 70   38.37   53.91    500.00   1.6493   270.00  19.7300    8.12   14.37   14.59  15.39  15.06   3.0575   1.0205    1.0000    0.9987
     0.00    0.00   -179.99  999.00    0.01   14.98   15.39   30.77   30.36  1.0026 -2.363E-06           1 -5.171E-04
 80   43.35   53.10    500.00   1.1089   270.00  19.8177   10.50   14.56   14.72  15.45  15.21   3.0386   1.0138    1.0000    0.9991
     0.00    0.00   -179.99  999.00    0.01   15.18   15.45   30.91   30.63  1.0018 -4.057E-06           1 -8.351E-04
 90   48.35   52.05    500.00   0.3169   270.00  19.9476   13.36   14.84   14.92  15.56  15.48   3.0110   1.0039    1.0000    0.9997
     0.00    0.00   -180.00  999.00    0.01   15.48   15.56   31.11   31.03  1.0006 -6.067E-06           1 -1.170E-03
100   53.37   50.72    500.00   0.0000   270.00  20.0000   14.94   14.94   15.00  15.60  15.60   2.9998   1.0000    1.0000    0.9999
     0.00    0.00   -180.00  999.00  999.00   15.60   15.60   31.19   31.19  1.0001 -3.845E-06           1  0.000E+00
110   58.39   49.38    500.00   0.0000   270.00  20.0000   14.94   14.94   15.00  15.60  15.60   2.9998   1.0000    1.0000    0.9999
     0.00    0.00   -180.00  999.00  999.00   15.60   15.60   31.19   31.19  1.0001 -3.845E-06           1  0.000E+00
                                                                        RAY REACHED GRID BOUNDARY
117   61.91   48.45    500.00   0.0000   270.00  20.0000   14.94   14.94   15.00  15.60  15.60   2.9998   1.0000    1.0000    0.9999
     0.00    0.00   -180.00  999.00  999.00   15.60   15.60   31.19   31.19  1.0001 -3.845E-06           1  0.000E+00
```

Figure (4-12). LISTING FOR RAY NUMBER 1 OF SAMPLE INPUT DATA FOR CURRENT GRID

```
PROJECT NO. PCURR ,  82/09/15    ,    PLOT NO.  1, PERIOD= 17.0SEC., RAY NO.  2, DELTAT= 25.00, CF=0.000000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS.  DEPTH,HGT(METER).  G,GR,GT,U,V,VT(METER/SECOND).

MAX  X        Y        DEPTH     CUR:SP   CUR:DI    PERIOD    RAY      PACK     WAVE      G         GR     HGT       KS          KF         KR
 ROTAT:D  PCT:D   ROTAT:C  PCT:CX  PCT:CY  GT        U         V        VT       BETA    DBETA/DT   BRK UP   NO   CURVATURE

   1   42.00     3.00   500.00   1.2800   270.00  17.5121   220.00  220.00  220.00  13.66  14.51   3.0000   1.0000   1.0000   1.0000
      0.00      0.00  -179.99   999.00     0.01   14.48    13.66    27.31   28.14  1.0000 -8.827E-05       1 -5.447E-03
  10   38.88     6.11   500.00   1.6060   270.00  17.6475   225.69  221.16  220.75  13.76  14.87   2.9923   0.9885   1.0000   1.0091
      0.00      0.00  -179.99   999.00     0.01   14.82    13.76    27.53   28.57  0.9821 -6.954E-05       1 -3.871E-03
  20   35.44     9.75   500.00   1.8520   270.00  17.7511   227.21  222.08  221.33  13.84  15.15   2.9876   0.9798   1.0000   1.0164
      0.00      0.00  -179.98   999.00     0.01   15.08    13.84    27.69   28.91  0.9680 -4.271E-05       1 -2.403E-03
  30   32.02    13.51   500.00   1.9795   270.00  17.8052   228.02  222.58  221.64  13.88  15.29   2.9844   0.9753   1.0000   1.0200
      0.00      0.00  -179.93   999.00     0.01   15.22    13.89    27.77   29.09  0.9611 -1.152E-05       1 -9.012E-04
  40   28.62    17.31   500.00   1.9904   270.00  17.8098   228.15  222.65  221.67  13.89  15.31   2.9813   0.9749   1.0000   1.0194
      0.00      0.00    -0.11   999.00     0.01   15.24    13.89    27.78   29.10  0.9624  2.136E-05       1  6.116E-04
  50   25.21    21.08   500.00   1.8851   270.00  17.7651   227.61  222.27  221.42  13.85  15.18   2.9781   0.9786   1.0000   1.0144
      0.00      0.00    -0.03   999.00     0.01   15.12    13.85    27.71   28.96  0.9717  5.302E-05       1  2.115E-03
  60   21.78    24.75   500.00   1.6621   270.00  17.6710   226.37  221.47  220.88  13.78  14.93   2.9764   0.9864   1.0000   1.0058
      0.00      0.00    -0.02   999.00     0.01   14.88    13.78    27.56   28.65  0.9886  8.072E-05       1  3.586E-03
  70   18.32    28.25   500.00   1.3180   270.00  17.5278   224.41  220.28  220.09  13.67  14.56   2.9785   0.9986   1.0000   0.9943
      0.00      0.00    -0.01   999.00     0.01   14.52    13.67    27.34   28.19  1.0116  1.022E-04       1  5.003E-03
  80   14.83    31.50   500.00   0.8488   270.00  17.3362   221.68  218.75  219.05  13.52  14.07   2.9877   1.0151   1.0000   0.9810
      0.00      0.00     0.00   999.00     0.01   14.05    13.52    27.04   27.57  1.0390  1.162E-04       1  6.336E-03
  90   11.29    34.43   500.00   0.2501   270.00  17.0977   218.09  216.93  217.79  13.33  13.48   3.0069   1.0363   1.0000   0.9672
      0.00      0.00     0.00   999.00     0.01   13.48    13.33    26.67   26.82  1.0690  1.224E-04       1  7.556E-03
 100    7.74    37.07   500.00   0.0000   270.00  17.0000   216.38  216.38  217.28  13.26  13.26   3.0211   1.0451   1.0000   0.9635
      0.00      0.00     0.00   999.00   999.00   13.26    13.26    26.52   26.52  1.0771  8.031E-05       1  0.000E+00
 110    4.18    39.69   500.00   0.0000   270.00  17.0000   216.38  216.38  217.28  13.26  13.26   3.0211   1.0451   1.0000   0.9635
      0.00      0.00     0.00   999.00   999.00   13.26    13.26    26.52   26.52  1.0771  8.031E-05       1  0.000E+00
                                                                              RAY REACHED GRID BOUNDARY
 118    1.33    41.79   500.00   0.0000   270.00  17.0000   216.38  216.38  217.28  13.26  13.26   3.0211   1.0451   1.0000   0.9635
      0.00      0.00     0.00   999.00   999.00   13.26    13.26    26.52   26.52  1.0771  8.031E-05       1  0.000E+00
```

Figure (4-13). LISTING FOR RAY NUMBER 2 OF SAMPLE INPUT DATA FOR CURRENT GRID

PROJ. NO. PDAC , 82/09/15
SCL = 1/310039 , CIN = 0
PLOT NO. 1 , DIR. = WAVPAK

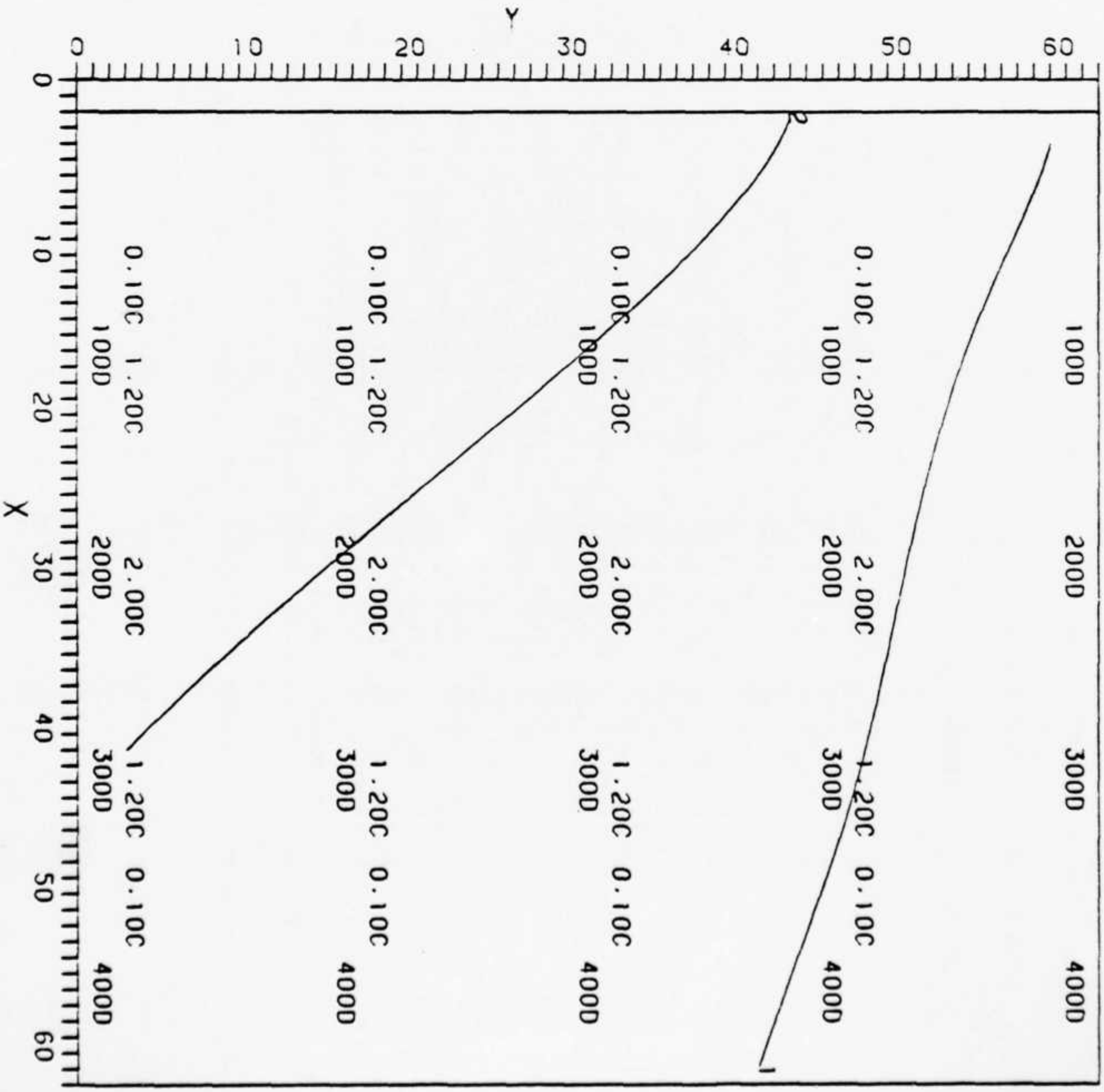

Figure (4-14). PLOT FOR SAMPLE INPUT DATA FOR WATER DEPTH AND CURRENT GRIDS

```
OJECT NO. PDAC  ,  82/09/15    ,      PLOT NO.  1, PERIOD= 20.0SEC., RAY NO.  1, DELTAT= 25.00, CF=0.100000, KRTOL=0.001000

E OUTPUT IS IN METRIC UNITS.  DEPTH,HGT(METER).  G,GR,GT,U,V,VT(METER/SECOND).

IX  X           Y        DEPTH     CUR:SF   CUR:DI    PERIOD     RAY      PACK     WAVE      G         GR      HGT          KS          KF         KR
ROTAT:D   PCT:D   ROTAT:C  PCT:CX   PCT:CY   GT        U         V         VT       BETA    DBETA/DT   BRK UF    NO    CURVATURE

  1    4.00    60.00     15.00   0.0000   270.00  20.0000    15.00    15.00    15.00  11.24  11.24   3.0000    1.0000    1.0000    1.0000
     0.00     0.01      0.00  999.00   999.00   11.24   11.24    11.82    11.82  1.0000 -2.198E-04            1 -5.478E-02
 10    8.10    58.54     45.75   0.0000   270.00  20.0000    22.55    22.68    24.66  16.73  16.73   2.2639    0.8195    0.9003    1.0229
     0.00     0.01      0.00  999.00   999.00   16.73   16.74    19.54    19.54  0.9558 -1.458E-04      BETA  2 -1.640E-02
 20   13.54    56.17     86.58   0.6460   270.00  19.7306    22.78    24.62    31.10  18.30  18.04   2.1619    0.7895    0.8860    1.0303
    -0.01     0.01      0.00  999.00     0.01   18.03   18.41    24.78    24.44  0.9421  1.482E-05            1  2.002E-03
 30   19.01    54.11    127.56   1.3958   270.00  19.4280    18.78    23.01    34.22  17.48  16.98   2.2114    0.8147    0.8826    1.0252
    -0.01     0.01     -0.01  999.00     0.01   16.93   17.81    27.44    26.66  0.9514  4.648E-05            1  6.345E-03
 40   24.21    52.56    166.55   1.8321   270.00  19.2553    14.93    21.09    35.68  16.27  15.71   2.2877    0.8483    0.8816    1.0197
    -0.02     0.01     -0.02  999.00     0.01   15.61   16.82    28.70    27.63  0.9618  3.373E-05            1  5.524E-03
 50   29.13    51.38    203.46   1.9961   270.00  19.1905    12.56    19.79    36.45  15.39  14.83   2.3482    0.8738    0.8813    1.0164
    -0.02     0.01     -0.16  999.00     0.01   14.71   16.06    29.30    28.12  0.9680  1.680E-05            1  3.416E-03
 60   33.87    50.36    239.04   1.9249   270.00  19.2179    11.84    19.10    37.06  14.88  14.36   2.3831    0.8881    0.8812    1.0151
    -0.02     0.01   -179.96  999.00     0.01   14.25   15.64    29.67    28.51  0.9706  4.102E-06            1  1.535E-03
 70   38.52    49.37    273.88   1.6370   270.00  19.3308    12.59    18.86    37.81  14.63  14.18   2.3956    0.8927    0.8812    1.0151
    -0.03     0.01   -179.99  999.00     0.01   14.10   15.47    29.99    28.99  0.9704 -5.142E-06            1  1.381E-04
 80   43.12    48.26    308.37   1.1396   270.00  19.5294    14.60    18.95    38.84  14.55  14.22   2.3914    0.8901    0.8812    1.0163
    -0.03     0.01   -179.99  999.00     0.01   14.18   15.47    30.37    29.66  0.9681 -1.262E-05            1 -8.899E-04
 90   47.70    46.94    342.76   0.4329   270.00  19.8186    17.66    19.30    40.28  14.58  14.45   2.3748    0.8821    0.8812    1.0184
    -0.03     0.01   -180.00  999.00     0.01   14.44   15.62    30.86    30.58  0.9641 -1.957E-05            1 -1.684E-03
100   52.30    45.36    377.21   0.0000   270.00  20.0000    19.47    19.46    41.08  14.60  14.60   2.3674    0.8774    0.8812    1.0207
    -0.04     0.01   -180.00  999.00   999.00   14.60   15.70    31.16    31.16  0.9598 -1.541E-05            1  3.407E-04
110   56.87    43.74    411.49   0.0000   270.00  20.0000    19.41    19.41    41.10  14.49  14.49   2.3788    0.8805    0.8812    1.0220
    -0.04     0.01   -180.00  999.00   999.00   14.49   15.60    31.19    31.19  0.9574 -1.599E-05            1  0.000E+00
120   61.42    42.14    445.67   0.0000   270.00  20.0000    19.41    19.41    41.10  14.49  14.49   2.3788    0.8805    0.8812    1.0220
    -0.04     0.01   -180.00  999.00   999.00   14.49   15.60    31.19    31.19  0.9574 -1.599E-05            1  0.000E+00
                                                                             RAY REACHED GRID BOUNDARY
121   61.88    41.98    449.08   0.0000   270.00  20.0000    19.41    19.41    41.10  14.49  14.49   2.3788    0.8805    0.8812    1.0220
    -0.05     0.01   -180.00  999.00   999.00   14.49   15.60    31.19    31.19  0.9574 -1.599E-05            1  0.000E+00
```

Figure (4-15). LISTING FOR RAY NUMBER 1 OF SAMPLE INPUT DATA FOR WATER DEPTH AND CURRENT GRIDS

```
PROJECT NO. PDAC  ,  82/09/15    ,    PLOT NO.  1, PERIOD= 17.0SEC., RAY NO.  2, DELTAT= 25.00, CF=0.000000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS.  DEPTH,HGT(METER).  G,GR,GT,U,V,VT(METER/SECOND).

MAX  X        Y        DEPTH    CUR:SP   CUR:DI    PERIOD    RAY     PACK     WAVE     G        GR      HGT        KS        KF        KR
 ROTAT:D  PCT:D   ROTAT:C  PCT:CX  PCT:CY  GT       U        V       VT       BETA    DBETA/DT  BRK UP  NO    CURVATURE

   1   42.00     3.00   299.99   1.2800   270.00  17.5121  220.00  220.00  220.00  13.66  14.51    3.0000    1.0000    1.0000    1.0000
    -0.03     0.01  -179.99  999.00     0.01  14.48   13.66   27.31   28.14  1.0000 -8.827E-05            1 -5.447E-03
  10   38.86     6.14   276.42   1.6076   270.00  17.6486  225.90  221.35  220.71  13.89  15.00    2.9794    0.9839    1.0000    1.0094
    -0.03     0.01  -179.99  999.00     0.01  14.96   13.89   27.48   28.53  0.9816 -7.526E-05            1 -4.852E-03
  20   35.39     9.84   250.44   1.8544   270.00  17.7527  227.67  222.64  221.22  14.08  15.40    2.9674    0.9716    1.0000    1.0181
    -0.02     0.01  -179.98  999.00     0.01  15.34   14.09   27.60   28.82  0.9648 -5.851E-05            1 -3.987E-03
  30   31.95    13.72   224.59   1.9809   270.00  17.8065  228.90  223.73  221.37  14.29  15.73    2.9558    0.9615    1.0000    1.0247
    -0.02     0.01  -179.92  999.00     0.01  15.66   14.31   27.60   28.91  0.9523 -4.162E-05            1 -3.260E-03
  40   28.50    17.73   198.76   1.9887   270.00  17.8100  229.67  224.63  221.12  14.54  16.00    2.9432    0.9531    1.0000    1.0293
    -0.02     0.01    -0.10  999.00     0.01  15.94   14.57   27.45   28.75  0.9439 -2.612E-05            1 -2.651E-03
  50   25.04    21.84   172.80   1.8770   270.00  17.7628  229.99  225.36  220.38  14.84  16.23    2.9288    0.9462    1.0000    1.0318
    -0.02     0.01    -0.03  999.00     0.01  16.17   14.90   27.10   28.32  0.9393 -1.020E-05            1 -2.050E-03
  60   21.54    26.01   146.58   1.6425   270.00  17.6642  229.87  225.89  219.07  15.18  16.40    2.9121    0.9409    1.0000    1.0317
    -0.01     0.01    -0.01  999.00     0.01  16.36   15.29   26.50   27.53  0.9395  1.461E-05            1 -1.181E-03
  70   17.99    30.18   119.90   1.2784   270.00  17.5124  229.12  226.03  216.97  15.51  16.45    2.8914    0.9388    1.0000    1.0266
    -0.01     0.01    -0.01  999.00     0.01  16.43   15.70   25.50   26.27  0.9488  6.661E-05            1  5.361E-04
  80   14.35    34.25    92.64   0.7757   270.00  17.3070  227.33  225.35  213.86  15.71  16.27    2.8637    0.9437    1.0000    1.0115
    -0.01     0.01     0.00  999.00     0.01  16.26   16.03   23.93   24.36  0.9774  1.746E-04            1  4.310E-03
  90   10.63    38.04    64.69   0.1041   270.00  17.0405  223.25  222.93  209.33  15.46  15.53    2.8360    0.9656    1.0000    0.9790
    -0.01     0.01     0.00  999.00   999.00  15.53   15.90   21.41   21.46  1.0435  3.457E-04      BETA   8  1.147E-02
 100    6.89    41.29    36.66   0.0000   270.00  17.0000  218.50  218.30  203.10  14.09  14.09    2.8212    1.0137    1.0000    0.9277
     0.00     0.01     0.00  999.00   999.00  14.09   14.60   17.33   17.33  1.1620  6.639E-04      BETA   2  2.947E-02
 110    3.51    43.49    11.31   0.0000   270.00  17.0000  205.08  204.83  193.16   9.52   9.52    3.1535    1.2332    1.0000    0.8524
     0.00     0.01     0.00  999.00   999.00   9.52    9.72   10.25   10.25  1.3762  9.723E-04      BETA   4  1.123E-01
                                                                                 RAY REACHED SHORE
 118    2.04    43.96     0.28   0.0000   270.00  17.0000  184.40  184.22  181.85   1.66   1.66    7.1769    2.9495    1.0000    0.8111
     0.00   999.00     0.00  999.00   999.00   1.66    1.67    1.67    1.67  1.5201  2.373E-04      BETA   8  9.645E-01
```

Figure (4-16). LISTING FOR RAY NUMBER 2 OF SAMPLE INPUT DATA FOR WATER DEPTH AND CURRENT GRIDS

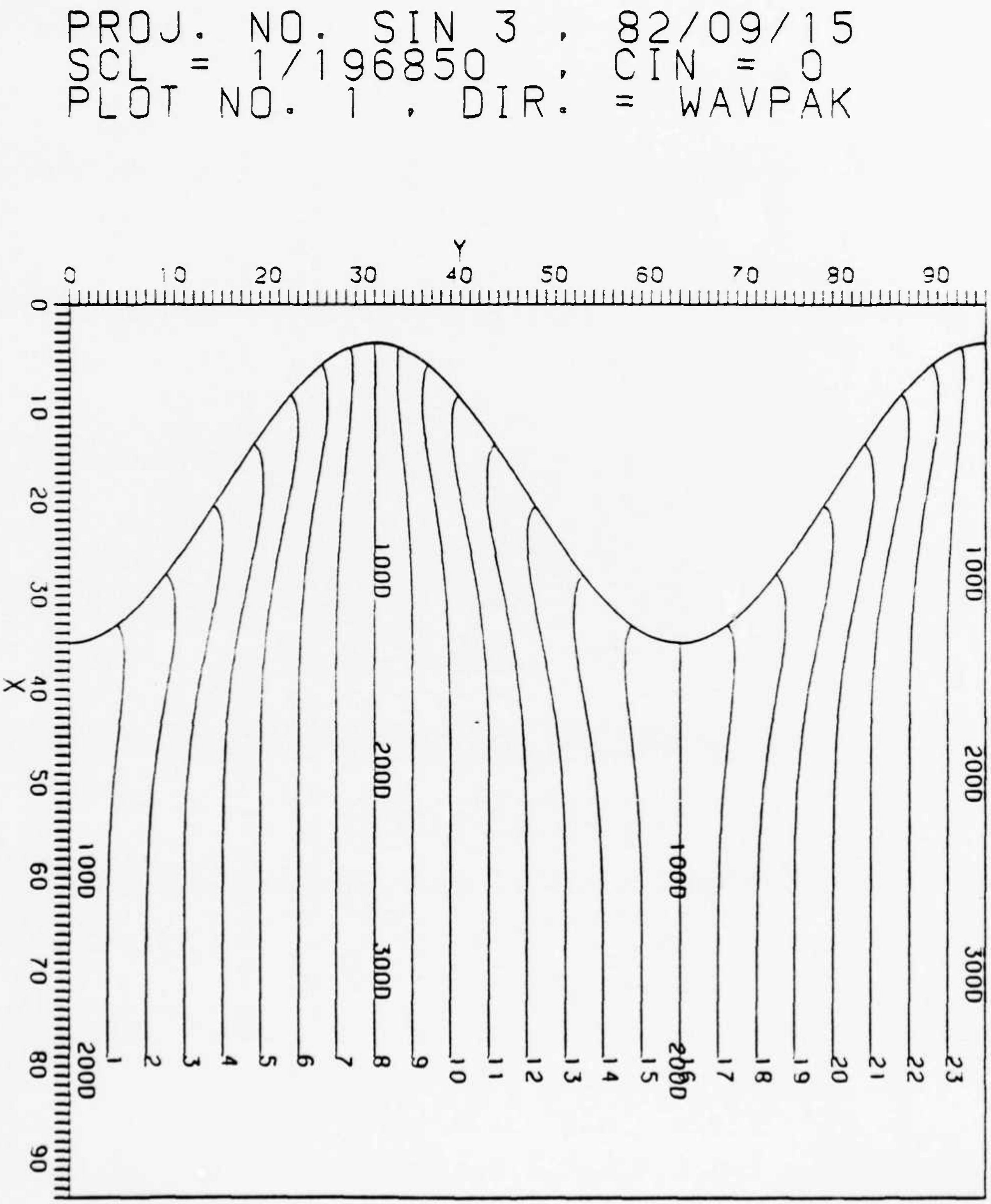

Figure (4-17). PLOT FOR SINUSOIDAL WATER DEPTH CONTOURS

```
PROJECT NO. SIN 3 ,  82/09/15    ,      PLOT NO.  1, PERIOD= 14.0SEC., RAY NO.  4, DELTAT= 10.00, CF=0.000000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS.  DEPTH,HGT(METER).  G,GR,GT,U,V,VT(METER/SECOND).

MAX  X        Y        DEPTH    CUR:SF   CUR:DI    PERIOD    RAY     PACK     WAVE     G        GR       HGT        KS         KF          KR
 ROTAT:D  FCT:D    ROTAT:C  FCT:CX   FCT:CY  GT       U         V       VT       BETA     DBETA/DT  BRK UP  NO     CURVATURE

  1   80.00   16.00   281.26   0.0000        0.00  14.0000   180.00  180.00  180.00  10.92  10.92    1.0000   1.0000     1.0000    1.0000
   57.53    0.01     0.00     0.00     0.00  10.92   10.92    21.84   21.84  1.0000  0.000E+00            1  0.000E+00
 20   73.36   16.00   250.12   0.0000        0.00  14.0000   180.00  180.00  180.00  10.92  10.92    1.0000   1.0000     1.0000    1.0000
   57.51    0.02     0.00     0.00     0.00  10.92   10.92    21.84   21.84  1.0000  0.000E+00            1  0.000E+00
 40   66.37   16.00   217.39   0.0000        0.00  14.0000   180.00  180.00  180.00  10.92  10.92    1.0000   1.0000     1.0000    1.0000
   57.38    0.02     0.00     0.00     0.00  10.92   10.92    21.84   21.84  1.0000  0.000E+00            1  0.000E+00
 60   59.37   16.00   184.62   0.0000        0.00  14.0000   180.18  180.18  179.91  10.99  10.99    0.9971   0.9967     1.0000    1.0004
   57.54    0.03     0.00     0.00     0.00  10.99   10.99    21.81   21.81  0.9992 -2.660E-05            1 -1.664E-03
 80   52.30   16.08   152.05   0.0000        0.00  14.0000   181.31  181.31  179.66  11.13  11.13    1.0005   0.9904     1.0000    1.0102
   57.54    0.03     0.00     0.00     0.00  11.13   11.13    21.75   21.75  0.9799 -1.907E-04            1 -4.454E-03
100   45.10   16.40   120.58   0.0000        0.00  14.0000   184.02  184.02  178.84  11.44  11.44    1.0219   0.9769     1.0000    1.0461
   57.50    0.02     0.00     0.00     0.00  11.44   11.49    21.55   21.55  0.9137 -4.834E-04            1 -8.902E-03
120   37.68   17.19    91.63   0.0000        0.00  14.0000   188.30  188.30  176.66  11.88  11.88    1.0740   0.9588     1.0000    1.1202
   57.22    0.04     0.00     0.00     0.00  11.88   12.13    20.99   20.99  0.7970 -6.077E-04            1 -9.938E-03
140   30.10   18.54    65.84   0.0000        0.00  14.0000   191.32  191.32  172.30  12.15  12.15    1.1230   0.9481     1.0000    1.1845
   56.75    0.03     0.00     0.00     0.00  12.15   12.85    19.75   19.75  0.7128 -1.185E-04            1 -2.182E-03
160   22.50   19.97    40.21   0.0000        0.00  14.0000   188.44  188.44  164.33  11.87  11.87    1.0655   0.9593     1.0000    1.1108
   55.37    0.17     0.00     0.00     0.00  11.87   13.00    17.11   17.11  0.8105  1.362E-03      BETA  2  1.979E-02
180   15.73   19.98     8.55   0.0000        0.00  14.0000   161.87  161.87  144.09   7.98   7.98    0.9446   1.1698     1.0000    0.8075
   55.46    2.70     0.00     0.00     0.00   7.98    8.38     8.89    8.89  1.5337  6.978E-03      BETA  4  2.179E-01
                                                                          RAY REACHED SHORE
187   14.75   19.44     0.20   0.0000        0.00  14.0000   130.35  130.35  127.26   1.40   1.40    1.9964   2.7917     1.0000    0.7151
   55.91    6.74     0.00     0.00     0.00   1.40    1.40     1.40    1.40  1.9554  2.248E-03      BETA 16  2.248E+00
```

Figure (4-18). LISTING FOR RAY NUMBER 4 IN FIGURE (4-17)

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PROJECT NO. SIN 3 ,  82/09/15    ,     PLOT NO.  1, PERIOD= 14.0SEC., RAY NO.  8, DELTAT= 10.00, CF=0.000000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS.  DEPTH,HGT(METER).  G,GR,GT,U,V,VT(METER/SECOND).

MAX  X         Y        DEPTH    CUR:SP   CUR:DI   PERIOD    RAY     PACK     WAVE     G        GR      HGT        KS          KF         KR
 ROTAT:D   PCT:D    ROTAT:C  PCT:CX   PCT:CY  GT      U        V       VT      BETA    DBETA/DT   BRK UP   NO   CURVATURE

   1   80.00   32.00   356.26   0.0000      0.00  14.0000  180.00  180.00  180.00  10.92  10.92   1.0000   1.0000   1.0000   1.0000
     -0.20    0.01     0.00     0.00    0.00   10.92  10.92   21.84   21.84  1.0000  0.000E+00         1  0.000E+00
  20   73.36   32.00   325.12   0.0000      0.00  14.0000  180.00  180.00  180.00  10.92  10.92   1.0000   1.0000   1.0000   1.0000
     -0.19    0.02     0.00     0.00    0.00   10.92  10.92   21.84   21.84  1.0000  0.000E+00         1  0.000E+00
  40   66.37   32.00   292.38   0.0000      0.00  14.0000  180.00  180.00  180.00  10.92  10.92   1.0000   1.0000   1.0000   1.0000
     -0.13    0.02     0.00     0.00    0.00   10.92  10.92   21.84   21.84  1.0000  0.000E+00         1  0.000E+00
  60   59.39   32.00   259.61   0.0000      0.00  14.0000  180.00  180.00  180.00  10.92  10.92   1.0000   1.0000   1.0000   1.0000
     -0.03    0.02     0.00     0.00    0.00   10.92  10.92   21.84   21.84  1.0000  0.000E+00         1  0.000E+00
  80   52.40   32.00   226.84   0.0000      0.00  14.0000  180.00  180.00  180.00  10.92  10.92   1.0000   1.0000   1.0000   1.0000
      0.10    0.01     0.00     0.00    0.00   10.92  10.92   21.84   21.84  1.0000  0.000E+00         1  0.000E+00
 100   45.41   32.00   194.11   0.0000      0.00  14.0000  180.00  180.00  180.00  10.97  10.97   0.9976   0.9976   1.0000   1.0000
      0.00    0.01     0.00     0.00    0.00   10.97  10.97   21.82   21.82  1.0000  0.000E+00         1  3.171E-08
 120   38.36   32.00   161.09   0.0000      0.00  14.0000  180.00  180.00  180.00  11.08  11.08   0.9949   0.9927   1.0000   1.0022
     -0.08    0.02     0.00     0.00    0.00   11.08  11.08   21.78   21.78  0.9956 -5.364E-05         1  3.233E-06
 140   31.20   32.00   127.51   0.0000      0.00  14.0000  180.00  180.00  180.00  11.38  11.38   0.9934   0.9793   1.0000   1.0143
     -0.04    0.03     0.00     0.00    0.00   11.38  11.38   21.62   21.62  0.9720 -2.088E-04         1  4.403E-06
 160   23.72   32.00    92.43   0.0000      0.00  14.0000  180.00  180.00  180.00  12.11  12.11   1.0029   0.9497   1.0000   1.0560
     -0.06    0.04     0.00     0.00    0.00   12.11  12.11   21.01   21.01  0.8967 -5.881E-04         1  1.018E-05
 180   15.66   32.00    54.65   0.0000      0.00  14.0000  180.00  180.00  180.00  13.06  13.06   1.0754   0.9143   1.0000   1.1762
     -0.03    0.07     0.00     0.00    0.00   13.06  13.06   18.83   18.83  0.7229 -1.106E-03         1  2.443E-06
 200    7.61   32.00    16.96   0.0000      0.00  14.0000  180.00  180.00  180.00  10.80  10.80   1.3603   1.0055   1.0000   1.3529
     -0.11    0.15     0.00     0.00    0.00   10.80  10.80   12.14   12.14  0.5463 -2.375E-04         1 -1.623E-04
                                                                               MAX= 215, PACKET CURVATURE AVERAGED
                                                                               RAY REACHED SHORE
 217    4.04   32.00     0.18   0.0000      0.00  14.0000  180.08  180.08  180.13   1.33   1.33   3.6478   2.8642   1.0000   1.2736
     -0.27  999.00     0.00     0.00    0.00    1.33   1.33    1.33    1.33  0.6165  3.320E-04   BETA  2 -4.338E-02
```

Figure (4-19). LISTING FOR RAY NUMBER 8 IN FIGURE (4-17)

PROJECT NO. 8IN 3 , 82/09/15 , PLOT NO. 1, PERIOD= 14.0SEC., RAY NO. 16, DELTAT= 10.00, CF=0.000000, KRTOL=0.001000

THE OUTPUT IS IN METRIC UNITS. DEPTH,HGT(METER). G,GR,GT,U,V,VT(METER/SECOND).

MAX	X	Y	DEPTH	CUR:SP	CUR:DI	PERIOD	RAY	PACK	WAVE	G	GR	HGT	KS	KF	KR
	ROTAT:D	PCT:D	ROTAT:C	FCT:CX	FCT:CY	GT	U	V	VT	BETA	DBETA/DT	BRK UP	NO	CURVATURE	
1	80.00	64.00	206.22	0.0000	0.00	14.0000	180.00	180.00	180.00	10.92	10.92	1.0000	1.0000	1.0000	1.0000
	-0.21	0.01	0.00	0.00	0.00	10.92	10.92	21.84	21.84	1.0000	0.000E+00		1	0.000E+00	
20	73.33	64.00	174.98	0.0000	0.00	14.0000	180.00	180.00	180.00	11.02	11.02	0.9948	0.9954	1.0000	0.9993
	0.00	0.01	0.00	0.00	0.00	11.02	11.02	21.80	21.80	1.0013	2.501E-05		1	-1.397E-08	
40	66.23	64.00	141.70	0.0000	0.00	14.0000	180.00	180.00	180.00	11.22	11.22	0.9793	0.9865	1.0000	0.9927
	0.01	0.01	0.00	0.00	0.00	11.22	11.22	21.71	21.71	1.0147	1.273E-04		1	-3.150E-07	
60	58.92	64.00	107.42	0.0000	0.00	14.0000	180.00	180.00	180.00	11.74	11.74	0.9348	0.9644	1.0000	0.9693
	0.01	0.02	0.00	0.00	0.00	11.74	11.74	21.36	21.36	1.0643	4.143E-04		1	-7.121E-07	
80	51.13	64.00	70.91	0.0000	0.00	14.0000	180.00	180.00	180.00	12.72	12.72	0.8432	0.9266	1.0000	0.9099
	-0.06	0.06	0.00	0.00	0.00	12.72	12.72	20.08	20.08	1.2077	1.077E-03		1	1.218E-05	
100	42.83	64.00	32.03	0.0000	0.00	14.0000	180.00	180.00	180.01	12.65	12.65	0.7683	0.9291	1.0000	0.8269
	0.01	0.05	0.00	0.00	0.00	12.65	12.65	15.76	15.76	1.4624	9.981E-04		1	2.193E-06	
120	36.54	64.00	2.53	0.0000	0.00	14.0000	180.03	180.03	180.04	4.85	4.85	1.3025	1.4998	1.0000	0.8684
	0.00	999.00	0.00	0.00	0.00	4.85	4.85	4.94	4.94	1.3259	-1.861E-03		1	4.432E-04	
										RAY REACHED SHORE					
125	36.02	64.00	0.11	0.0000	0.00	14.0000	180.03	180.03	180.03	1.05	1.05	2.8826	3.2314	1.0000	0.8921
	-0.04	999.00	0.00	0.00	0.00	1.05	1.05	1.05	1.05	1.2566	-6.322E-04	BETA	2	-2.604E-03	

Figure (4-20). LISTING FOR RAY NUMBER 16 IN FIGURE (4-17)

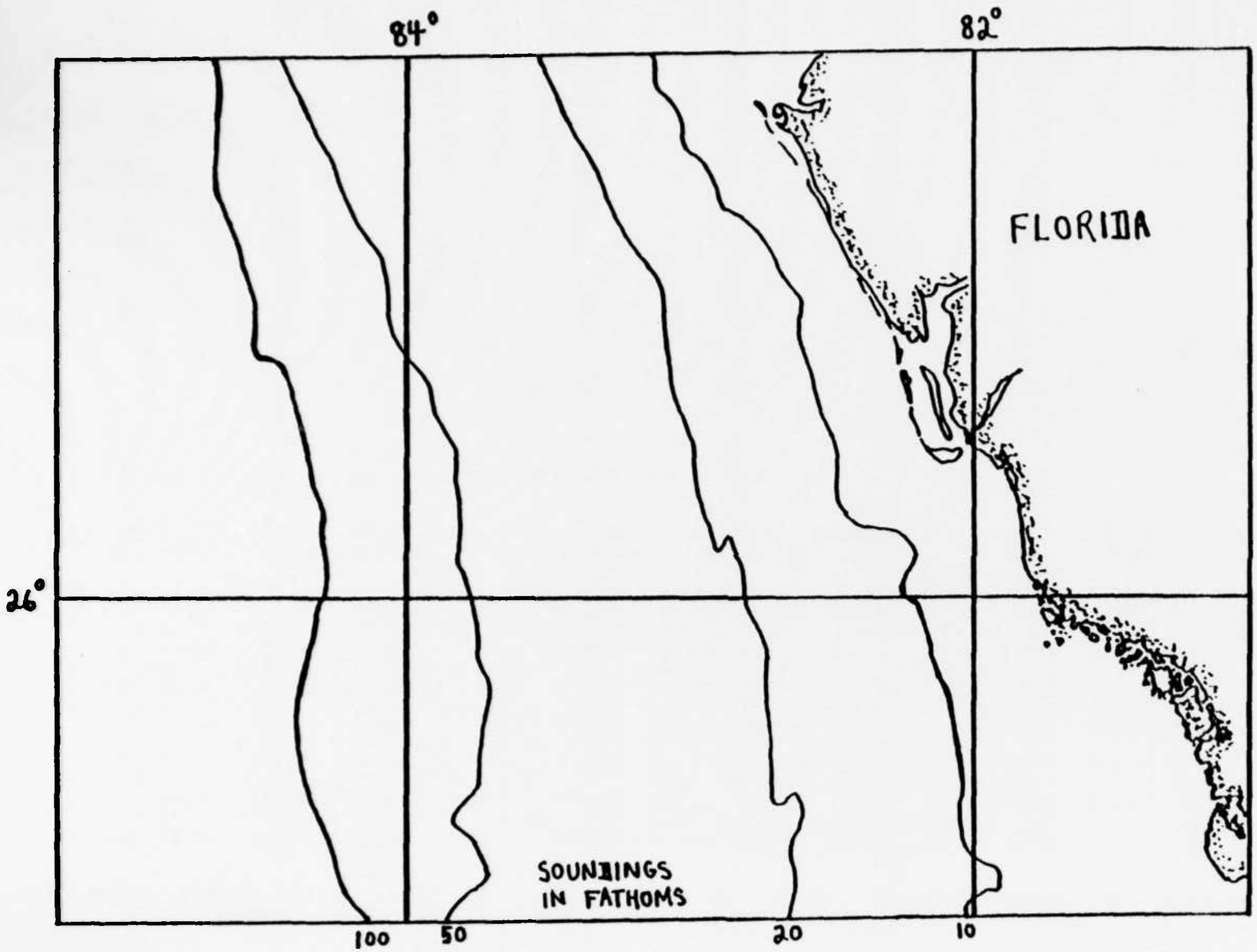

Figure (4-21). GULF OF MEXICO OFF THE SOUTHWESTERN FLORIDA COAST

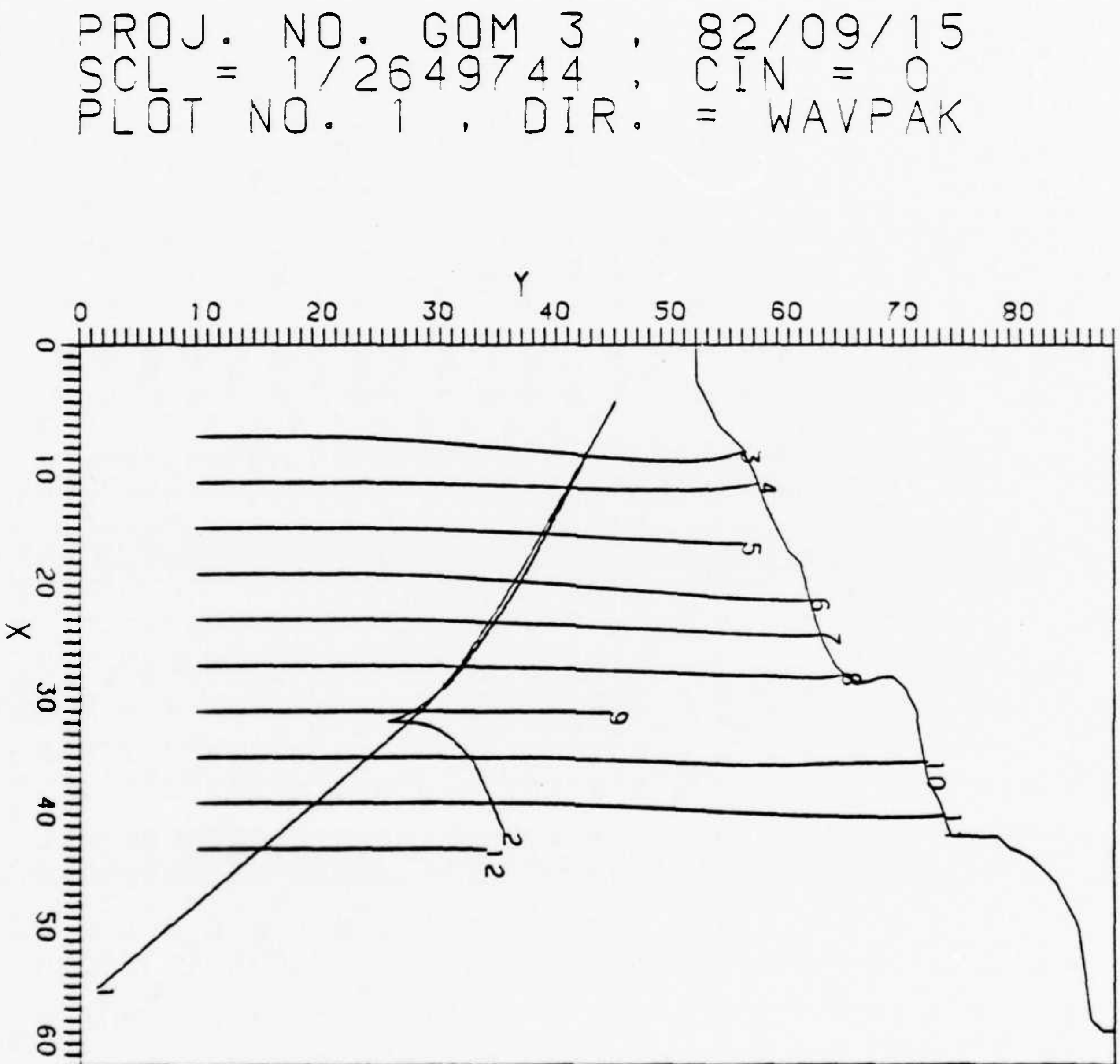

Figure (4-22). PLOT OF RAYS OFF THE SOUTHWESTERN FLORIDA COAST

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THE OUTPUT IS IN ENGLISH UNITS.  DEPTH,HGT(FEET).  G,GR,GT,U,V,VT(FEET/SECOND).

MAX   X        Y        DEPTH     CUR:SF    CUR:DI     PERIOD     RAY      PACK      WAVE      G        GR       HGT        KS           KF          KR
 ROTAT:D   PCT:D    ROTAT:C   PCT:CX   PCT:CY   GT       U          V        VT        BETA     DBETA/DT   BRK UP    NO    CURVATURE

   1    5.00   46.00     74.74   0.0000   270.00  12.0000    30.00    30.00    30.00  35.42  35.42    1.0000    1.0000     1.0000    1.0000
    -74.29   11.91     0.00     0.00     0.00  35.42   35.42    43.83    43.83  1.0000 -4.694E-05           1   2.022E-02
  25    7.55   44.57     98.89   0.0000   270.00  12.0000    28.73    28.73    24.50  36.56  36.56    0.9862    0.9843     0.9983    1.0037
    -62.56    4.82     0.00     0.00     0.00  36.56   36.66    48.50    48.50  0.9927  1.372E-05           1   2.641E-03
  50   10.26   43.10    116.75   0.0000   270.00  12.0000    28.62    28.62    23.09  36.72  36.72    0.9718    0.9822     0.9972    0.9922
    -44.88    1.82     0.00     0.00     0.00  36.72   36.89    51.17    51.17  1.0159  2.018E-05           1  -1.622E-04
  75   12.96   41.62    135.50   0.0000   270.00  12.0000    28.73    28.73    22.02  36.48  36.48    0.9627    0.9854     0.9966    0.9802
    -32.19    2.32     0.00     0.00     0.00  36.48   36.73    53.43    53.43  1.0407  1.871E-05           1  -1.242E-04
 100   15.63   40.15    147.78   0.0000   270.00  12.0000    28.97    28.97    21.33  36.15  36.15    0.9564    0.9898     0.9961    0.9699
    -58.22    1.97     0.00     0.00     0.00  36.15   36.48    54.65    54.65  1.0629  2.003E-05           1  -3.163E-03
 125   18.25   38.68    164.91   0.0000   270.00  12.0000    29.58    29.58    20.13  35.52  35.52    0.9534    0.9987     0.9958    0.9587
    -68.34    1.61     0.00     0.00     0.00  35.52   36.01    56.09    56.09  1.0879  1.716E-05           1  -5.144E-03
 150   20.81   37.21    181.92   0.0000   270.00  12.0000    30.44    30.44    18.98  34.75  34.75    0.9591    1.0096     0.9956    0.9542
    -65.48    2.34     0.00     0.00     0.00  34.75   35.46    57.24    57.24  1.0983 -2.225E-06           1  -6.375E-03
 175   23.28   35.71    193.73   0.0000   270.00  12.0000    32.46    32.46    17.43  33.86  33.86    0.9734    1.0227     0.9954    0.9561
    -81.77    0.54     0.00     0.00     0.00  33.86   35.06    57.90    57.90  1.0940  3.927E-06           1  -1.097E-02
 200   25.63   34.18    210.25   0.0000   270.00  12.0000    33.49    33.49    16.52  33.01  33.01    0.9856    1.0360     0.9953    0.9559
    -64.23    1.06     0.00     0.00     0.00  33.01   34.51    58.68    58.68  1.0944 -3.582E-06           1  -5.104E-03
 225   27.90   32.63    228.96   0.0000   270.00  12.0000    35.38    35.38    15.57  31.90  31.90    0.9891    1.0537     0.9952    0.9432
    -78.95    2.03     0.00     0.00     0.00  31.90   33.91    59.37    59.37  1.1241  5.742E-05           1  -2.039E-02
 250   29.94   31.06    269.76   0.0000   270.00  12.0000    40.14    40.14    13.25  29.26  29.26    0.9905    1.1003     0.9952    0.9045
    -79.85    1.46     0.00     0.00     0.00  29.26   32.81    60.37    60.37  1.2222  7.219E-05           1  -3.524E-02
 275   31.68   29.49    325.17   0.0000   270.00  12.0000    44.18    44.18    11.63  26.81  26.81    1.0109    1.1495     0.9952    0.8837
    -83.73    0.52     0.00     0.00     0.00  26.81   31.80    61.03    61.03  1.2806  2.343E-05           1  -2.706E-02
 300   33.20   27.93    373.55   0.0000   270.00  12.0000    47.07    47.07    10.81  25.24  25.24    1.0370    1.1846     0.9952    0.8796
    -87.18    0.42     0.00     0.00     0.00  25.24   31.31    61.29    61.29  1.2924 -1.003E-06           1  -2.007E-02
 325   34.60   26.38    423.28   0.0000   270.00  12.0000    48.57    48.57    10.42  24.40  24.40    1.0579    1.2049     0.9952    0.8822
    -82.61    0.56     0.00     0.00     0.00  24.40   31.03    61.41    61.41  1.2848 -9.049E-06           1  -7.437E-03
 350   35.93   24.85    483.22   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
    -89.25    0.84     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
 375   37.24   23.34    534.64   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
    -96.83    0.40     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
 400   38.55   21.82    566.18   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
   -102.41    0.31     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
 425   39.85   20.30    609.05   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
   -107.90    1.12     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
 450   41.16   18.79    647.32   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
    -94.22    0.84     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
 475   42.47   17.27    694.47   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
    -83.71    1.33     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
 500   43.78   15.76    698.65   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
    -65.92    7.81     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
 525   45.09   14.24   1128.34   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
    -74.69    5.06     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
 550   46.40   12.72   2130.94   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
    -61.16    3.62     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
 575   47.70   11.21   3487.60   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
    -50.50    0.82     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
 600   49.01    9.69   5199.19   0.0000   270.00  12.0000    49.21    49.21    10.08  23.85  23.85    1.0742    1.2187     0.9952    0.8857
    -52.55    3.53     0.00     0.00     0.00  23.85   30.75    61.50    61.50  1.2746 -9.920E-06           1   0.000E+00
```

Figure (4-23). PARTIAL LISTING FOR RAY NUMBER 1 IN FIGURE (4-22)

PROJECT NO. GOM 3 , 82/09/15 , PLOT NO. 1, PERIOD= 12.0SEC., RAY NO. 2, DELTAT= 50.00, CF=0.005000, KRTOL=0.00[illegible]000

THE OUTPUT IS IN ENGLISH UNITS. DEPTH,HGT(FEET). G,GR,GT,U,V,VT(FEET/SECOND).

MAX	X	Y	DEPTH	CUR:SP	CUR:DI	PERIOD	RAY	PACK	WAVE	G	GR	HGT	KS	KF	KR
ROTAT:D	PCT:D	ROTAT:C	PCT:CX	PCT:CY	GT	U	V	VT	BETA	DBETA/DT	BRK UP	NO	CURVATURE		
1	5.00	46.00	74.74	0.0000	270.00	12.0000	28.00	28.00	28.00	35.42	35.42	1.0000	1.0000	1.0000	1.0000
-74.29	11.91	0.00	0.00	0.00	35.42	35.42	43.83	43.83	1.0000	-5.211E-05		1	2.094E-02		
25	7.60	44.67	98.35	0.0000	270.00	12.0000	26.66	26.66	21.95	36.52	36.52	0.9878	0.9848	0.9983	1.0048
-63.41	4.82	0.00	0.00	0.00	36.52	36.65	48.41	48.41	0.9905	1.516E-05		1	2.796E-03		
50	10.35	43.29	116.22	0.0000	270.00	12.0000	26.55	26.55	20.13	36.66	36.66	0.9715	0.9830	0.9972	0.9911
-50.16	1.82	0.00	0.00	0.00	36.66	36.89	51.10	51.10	1.0181	2.530E-05		1	-3.778E-04		
75	13.09	41.92	136.18	0.0000	270.00	12.0000	26.78	26.78	18.39	36.32	36.32	0.9608	0.9875	0.9966	0.9763
-40.35	2.22	0.00	0.00	0.00	36.32	36.72	53.50	53.50	1.0492	2.020E-05		1	-6.765E-04		
100	15.80	40.54	146.18	0.0000	270.00	12.0000	27.11	27.11	17.57	36.01	36.01	0.9539	0.9918	0.9961	0.9655
-54.81	1.97	0.00	0.00	0.00	36.01	36.51	54.50	54.50	1.0727	1.973E-05		1	-2.998E-03		
125	18.46	39.16	161.05	0.0000	270.00	12.0000	28.08	28.08	16.26	35.36	35.36	0.9521	1.0009	0.9958	0.9552
-77.45	0.96	0.00	0.00	0.00	35.36	36.12	55.79	55.79	1.0960	1.679E-05		1	-1.044E-02		
150	21.02	37.74	179.34	0.0000	270.00	12.0000	30.10	30.10	13.90	34.14	34.14	0.9679	1.0187	0.9955	0.9544
-74.79	2.78	0.00	0.00	0.00	34.14	35.55	57.08	57.08	1.0978	-1.921E-05		1	-1.572E-02		
175	23.41	36.27	188.97	0.0000	270.00	12.0000	33.17	33.17	12.36	32.92	32.92	0.9981	1.0372	0.9954	0.9667
-84.75	2.60	0.00	0.00	0.00	32.92	35.22	57.65	57.65	1.0701	-2.584E-06		1	-1.895E-02		
200	25.64	34.74	206.33	0.0000	270.00	12.0000	35.12	35.12	10.68	31.53	31.53	1.0190	1.0599	0.9952	0.9660
-70.13	1.06	0.00	0.00	0.00	31.53	34.64	58.51	58.51	1.0715	-6.567E-06		1	-1.084E-02		
225	27.76	33.22	220.64	0.0000	270.00	12.0000	36.58	36.58	9.69	30.48	30.48	1.0421	1.0781	0.9951	0.9713
-68.83	1.97	0.00	0.00	0.00	30.48	34.17	59.08	59.08	1.0599	3.647E-06		1	-1.424E-02		
250	29.63	31.68	251.56	0.0000	270.00	12.0000	44.45	44.45	6.63	26.27	26.27	1.0922	1.1611	0.9951	0.9453
-83.57	1.46	0.00	0.00	0.00	26.27	33.26	59.99	59.99	1.1190	8.305E-05		1	-9.005E-02		
275	30.93	30.20	302.28	0.0000	270.00	12.0000	52.15	52.15	3.46	21.22	21.22	1.1791	1.2919	0.9951	0.9172
-81.88	1.06	0.00	0.00	0.00	21.22	32.15	60.83	60.83	1.1888	2.915E-05		1	-6.368E-02		
300	31.84	28.88	344.19	0.0000	270.00	12.0000	58.42	58.42	1.56	17.26	17.26	1.3047	1.4327	0.9951	0.9152
-83.66	0.24	0.00	0.00	0.00	17.26	31.57	61.16	61.16	1.1938	2.173E-05		1	-6.265E-02		
325	32.47	27.75	380.20	0.0000	270.00	12.0000	64.77	64.77	0.12	13.38	13.38	1.4816	1.6269	0.9951	0.9152
-86.58	0.40	0.00	0.00	0.00	13.38	31.26	61.31	61.31	1.1938	2.173E-05		1	-1.381E-01		
350	32.79	26.91	403.51	0.0000	270.00	12.0000	74.64	74.64	358.90	7.67	7.67	1.9576	2.1495	0.9951	0.9152
-87.65	0.79	0.00	0.00	0.00	7.67	31.12	61.37	61.37	1.1938	2.173E-05		1	-4.632E-01		
375	32.86	26.55	414.57	0.0000	270.00	12.0000	83.62	83.62	357.75	2.24	2.24	3.6253	3.9807	0.9951	0.9152
-87.08	0.79	0.00	0.00	0.00	2.24	31.07	61.39	61.39	1.1938	2.173E-05		1	-3.931E-01		
383	32.87	26.50	416.13	0.0000	270.00	12.0000	84.78	84.78	357.50	1.48	1.48	4.4597	4.8969	0.9951	0.9152
-86.99	0.79	0.00	0.00	0.00	1.48	31.06	61.40	61.40	1.1938	2.173E-05		1	-4.182E-01		
MAX = 383,	REFLECTION:	NEAR REFLECTION POINT													
400	32.87	26.61	412.49	0.0000	270.00	12.0000	270.74	270.74	356.06	2.53	2.53	3.4055	3.7395	0.9951	0.9152
-87.23	0.79	0.00	0.00	0.00	2.53	31.07	61.39	61.39	1.1938	-2.173E-05		1	-1.337E-01		
425	32.87	26.87	404.81	0.0000	270.00	12.0000	271.93	271.93	355.59	3.44	3.44	2.9232	3.2098	0.9950	0.9152
-87.80	0.79	0.00	0.00	0.00	3.44	31.11	61.37	61.37	1.1938	-2.173E-05		1	-4.956E-02		
450	32.89	27.18	397.39	0.0000	270.00	12.0000	272.82	272.82	355.24	4.11	4.11	2.6741	2.9364	0.9950	0.9152
-87.94	0.40	0.00	0.00	0.00	4.11	31.15	61.36	61.36	1.1938	-2.173E-05		1	-4.306E-02		
475	32.91	27.55	386.52	0.0000	270.00	12.0000	273.51	273.51	354.79	4.73	4.73	2.4925	2.7369	0.9950	0.9152
-88.78	0.40	0.00	0.00	0.00	4.73	31.22	61.33	61.33	1.1938	-2.173E-05		1	-2.378E-02		
500	32.93	27.97	374.03	0.0000	270.00	12.0000	273.97	273.97	354.35	5.23	5.23	2.3703	2.6027	0.9950	0.9152
-89.71	0.40	0.00	0.00	0.00	5.23	31.30	61.29	61.29	1.1938	-2.173E-05		1	-1.515E-02		
525	32.98	28.47	359.02	0.0000	270.00	12.0000	276.31	276.31	353.38	7.03	7.03	2.0435	2.2439	0.9950	0.9152
-86.67	0.46	0.00	0.00	0.00	7.03	31.43	61.23	61.23	1.1938	-2.173E-05		1	-1.081E-01		
550	33.09	29.20	338.41	0.0000	270.00	12.0000	282.14	282.14	351.79	11.00	11.00	1.6340	1.7943	0.9950	0.9152
-85.72	0.61	0.00	0.00	0.00	11.00	31.64	61.13	61.13	1.1938	-2.173E-05		1	-1.734E-01		
575	33.46	30.34	303.66	0.0000	270.00	12.0000	293.66	293.66	348.97	18.29	18.29	1.2674	1.3917	0.9950	0.9152

Figure (4-24). PARTIAL LISTING FOR RAY NUMBER 2 IN FIGURE (4-22)

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PROJECT NO. GOM 3 ,  82/09/15    ,     PLOT NO.  1, PERIOD= 12.0SEC., RAY NO.  6, DELTAT= 50.00, CF=0.005000, KRTOL=0.001000

THE OUTPUT IS IN ENGLISH UNITS.  DEPTH,HGT(FEET).  G,GR,GT,U,V,VT(FEET/SECOND).

MAX  X        Y        DEPTH    CUR:SP   CUR:DI    PERIOD    RAY     PACK    WAVE     G        GR      HGT       KS         KF         KR
 ROTAT:D  PCT:D    ROTAT:C  PCT:CX   PCT:CY  GT      U        V       VT      BETA    DBETA/DT   BRK UP   NO   CURVATURE

   1   20.00   10.00   877.93   0.0000   270.00  12.0000   270.00  270.00  270.00  30.75  30.75    1.0000    1.0000    1.0000    1.0000
    -84.19    6.75     0.00     0.00     0.00  30.75  30.75    61.50   61.50  1.0000  0.000E+00           1   0.000E+00
  25   20.00   12.48   716.56   0.0000   270.00  12.0000   270.00  270.00  270.00  30.75  30.75    1.0000    1.0000    1.0000    1.0000
    -70.57    0.35     0.00     0.00     0.00  30.75  30.75    61.50   61.50  1.0000  0.000E+00           1   0.000E+00
  50   20.00   15.06   579.74   0.0000   270.00  12.0000   270.00  270.00  270.00  30.75  30.75    1.0000    1.0000    1.0000    1.0000
    -76.30    1.12     0.00     0.00     0.00  30.75  30.75    61.50   61.50  1.0000  0.000E+00           1   0.000E+00
  75   20.00   17.64   528.04   0.0000   270.00  12.0000   270.00  270.00  270.00  30.75  30.75    1.0000    1.0000    1.0000    1.0000
    -52.97    0.52     0.00     0.00     0.00  30.75  30.75    61.50   61.50  1.0000  0.000E+00           1   0.000E+00
 100   20.00   20.23   464.00   0.0000   270.00  12.0000   270.01  270.01  269.98  30.90  30.90    0.9975    0.9975    1.0000    1.0000
    -61.91    0.44     0.00     0.00     0.00  30.90  30.90    61.45   61.45  1.0000 -1.346E-07           1  -1.036E-03
 125   20.00   22.83   396.69   0.0000   270.00  12.0000   270.23  270.23  269.93  31.15  31.15    0.9941    0.9935    1.0000    1.0007
    -61.50    0.53     0.00     0.00     0.00  31.15  31.15    61.36   61.36  0.9987 -1.763E-06           1  -2.624E-03
 150   20.03   25.47   314.40   0.0000   270.00  12.0000   270.99  270.99  269.72  31.95  31.95    0.9834    0.9810    1.0000    1.0024
    -60.03    1.08     0.00     0.00     0.00  31.95  31.96    60.95   60.95  0.9953 -5.162E-06           1  -7.984E-03
 175   20.10   28.19   269.45   0.0000   270.00  12.0000   272.09  272.09  269.37  32.78  32.78    0.9736    0.9686    1.0000    1.0052
    -57.22    0.92     0.00     0.00     0.00  32.78  32.81    60.36   60.36  0.9897 -1.269E-06           1  -5.755E-03
 200   20.23   30.98   233.19   0.0000   270.00  12.0000   272.96  272.96  268.98  33.70  33.70    0.9611    0.9552    0.9999    1.0062
    -63.39    1.47     0.00     0.00     0.00  33.70  33.78    59.50   59.50  0.9877  2.522E-07           1  -6.217E-03
 225   20.40   33.84   201.57   0.0000   270.00  12.0000   273.70  273.70  268.58  34.66  34.66    0.9440    0.9419    0.9999    1.0023
    -74.82    1.72     0.00     0.00     0.00  34.66  34.80    58.29   58.29  0.9954  6.760E-06           1  -3.106E-03
 250   20.60   36.77   183.75   0.0000   270.00  12.0000   274.28  274.28  268.11  35.19  35.19    0.9338    0.9347    0.9997    0.9993
    -55.30    1.83     0.00     0.00     0.00  35.19  35.40    57.35   57.35  1.0014  5.932E-07           1  -3.340E-03
 275   20.83   39.74   158.15   0.0000   270.00  12.0000   274.58  274.58  267.99  35.97  35.97    0.9249    0.9246    0.9995    1.0008
    -94.56    1.96     0.00     0.00     0.00  35.97  36.20    55.56   55.56  0.9985 -6.272E-06           1  -1.655E-06
 300   21.08   42.77   138.94   0.0000   270.00  12.0000   274.61  274.61  268.10  36.43  36.43    0.9226    0.9187    0.9991    1.0051
    -87.52    0.53     0.00     0.00     0.00  36.43  36.67    53.79   53.79  0.9898 -5.594E-06           1  -3.778E-04
 325   21.32   45.83   121.64   0.0000   270.00  12.0000   274.55  274.55  268.76  36.69  36.69    0.9197    0.9154    0.9986    1.0061
   -109.23    1.13     0.00     0.00     0.00  36.69  36.88    51.81   51.81  0.9878  1.236E-06           1   3.220E-04
 350   21.57   48.91   109.17   0.0000   270.00  12.0000   274.53  274.53  268.94  36.67  36.67    0.9184    0.9157    0.9977    1.0053
    -90.55    0.57     0.00     0.00     0.00  36.67  36.85    50.10   50.10  0.9896  1.272E-06           1   1.108E-04
 375   21.81   51.96    87.81   0.0000   270.00  12.0000   274.42  274.42  268.93  36.09  36.09    0.9279    0.9230    0.9963    1.0090
    -71.53    3.28     0.00     0.00     0.00  36.09  36.26    46.53   46.53  0.9823 -1.764E-05           1   2.299E-03
 400   22.04   54.96    77.20   0.0000   270.00  12.0000   274.12  274.12  268.54  35.44  35.44    0.9391    0.9314    0.9943    1.0140
    -66.53    6.55     0.00     0.00     0.00  35.44  35.61    44.37   44.37  0.9727  7.376E-06           1   7.729E-03
 425   22.22   57.83    54.54   0.0000   270.00  12.0000   273.16  273.16  267.47  32.91  32.91    0.9640    0.9667    0.9906    1.0068
    -87.72    1.68     0.00     0.00     0.00  32.91  33.07    38.65   38.65  0.9866 -8.963E-06           1   1.790E-03
 450   22.33   60.53    42.81   0.0000   270.00  12.0000   270.95  270.95  265.22  30.69  30.69    0.9855    1.0009    0.9841    1.0006
    -68.96    8.76     0.00     0.00     0.00  30.69  30.85    34.86   34.86  0.9988  4.989E-05           1   1.854E-02
 475   22.32   62.76    10.01   0.0000   270.00  12.0000   268.94  268.94  266.04  17.18  17.18    1.2562    1.3379    0.9521    0.9862
    -88.34  999.00     0.00     0.00     0.00  17.18  17.20    17.70   17.70  1.0283 -1.453E-05           1   1.343E-02
                                                                                 RAY REACHED SHORE
 480   22.32   62.99     3.31   0.0000   270.00  12.0000   268.80  268.80  267.13  10.17  10.17    1.4843    1.7390    0.8638    0.9881
    -88.73  999.00     0.00     0.00     0.00  10.17  10.17    10.27   10.27  1.0243 -1.559E-05           1   5.251E-03
```

Figure (4-25). LISTING FOR RAY NUMBER 6 IN FIGURE (4-22)

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PROJECT NO. GOM 3 ,  82/09/15    ,     PLOT NO.  1, PERIOD= 12.0SEC., RAY NO. 12, DELTAT= 50.00, CF=0.005000, KRTOL=0.001000

THE OUTPUT IS IN ENGLISH UNITS.  DEPTH,HGT(FEET).  G,GR,GT,U,V,VT(FEET/SECOND).

MAX  X        Y        DEPTH    CUR:SF   CUR:DI   PERIOD    RAY      PACK    WAVE     G        GR     HGT       KS         KF        KR
 ROTAT:D  PCT:D    ROTAT:C  PCT:CX   PCT:CY  GT       U         V       VT      BETA     DBETA/DT  BRK UP   NO   CURVATURE

   1   44.00   10.00  3720.80   0.0000    270.00  12.0000  270.00  270.00  270.00  30.75  30.75   1.0000   1.0000    1.0000   1.0000
    -92.34   1.76     0.00    0.00     0.00  30.75   30.75   61.50   61.50  1.0000  0.000E+00            1  0.000E+00
  25   44.00   12.48  1907.92   0.0000    270.00  12.0000  270.00  270.00  270.00  30.75  30.75   1.0000   1.0000    1.0000   1.0000
    -82.21   2.68     0.00    0.00     0.00  30.75   30.75   61.50   61.50  1.0000  0.000E+00            1  0.000E+00
  50   44.00   15.06   818.87   0.0000    270.00  12.0000  270.00  270.00  270.00  30.75  30.75   1.0000   1.0000    1.0000   1.0000
    -73.19   6.49     0.00    0.00     0.00  30.75   30.75   61.50   61.50  1.0000  0.000E+00            1  0.000E+00
  75   44.00   17.64   672.75   0.0000    270.00  12.0000  270.00  270.00  270.00  30.75  30.75   1.0000   1.0000    1.0000   1.0000
   -107.89   1.15     0.00    0.00     0.00  30.75   30.75   61.50   61.50  1.0000  0.000E+00            1  0.000E+00
 100   44.00   20.22   561.81   0.0000    270.00  12.0000  270.00  270.00  270.00  30.75  30.75   1.0000   1.0000    1.0000   1.0000
   -106.21   1.12     0.00    0.00     0.00  30.75   30.75   61.50   61.50  1.0000  0.000E+00            1  0.000E+00
 125   44.00   22.81   519.07   0.0000    270.00  12.0000  270.00  270.00  270.00  30.75  30.75   1.0000   1.0000    1.0000   1.0000
    -84.15   0.61     0.00    0.00     0.00  30.75   30.75   61.50   61.50  1.0000  0.000E+00            1  0.000E+00
 150   44.00   25.39   471.15   0.0000    270.00  12.0000  270.00  270.00  269.99  30.89  30.89   0.9977   0.9977    1.0000   1.0000
    -80.10   0.16     0.00    0.00     0.00  30.89   30.89   61.46   61.46  1.0000  0.000E+00            1 -2.862E-04
 175   44.00   27.99   404.71   0.0000    270.00  12.0000  270.08  270.08  269.98  31.11  31.11   0.9943   0.9941    1.0000   1.0001
    -82.51   0.60     0.00    0.00     0.00  31.11   31.11   61.37   61.37  0.9997 -7.882E-07            1 -6.285E-04
 200   44.01   30.62   330.16   0.0000    270.00  12.0000  270.23  270.23  269.94  31.74  31.74   0.9852   0.9843    1.0000   1.0009
    -80.20   0.76     0.00    0.00     0.00  31.74   31.74   61.07   61.07  0.9982 -1.291E-06            1 -1.863E-03
 225   44.03   33.33   259.30   0.0000    270.00  12.0000  270.79  270.79  269.73  33.05  33.05   0.9637   0.9645    1.0000   0.9992
    -73.26   4.78     0.00    0.00     0.00  33.05   33.06   60.16   60.16  1.0016  4.581E-06            1 -5.721E-03
 239   44.06   34.91   232.41   0.0000    270.00  12.0000  271.22  271.22  269.48  33.79  33.79   0.9526   0.9539    1.0000   0.9986
    -26.55   3.36     0.00    0.00     0.00  33.79   33.81   59.48   59.48  1.0028 -2.344E-06            1 -2.214E-03
MAX = 239,     REFLECTION:   SNELLS LAW WITH PHASE VELOCITY
                                                                            REFLECTION HANG-UP
 239   44.06   34.91   232.41   0.0000     10.14  12.0000  300.36  300.36   40.03  33.79  33.79   0.9526   0.9539    1.0000   0.9986
    -26.55   3.36     0.00    0.00     0.00  33.79   33.81   59.48   59.48  1.0028 -2.344E-06            1 -2.214E-03
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Figure (4-26). LISTING FOR RAY NUMBER 12 IN FIGURE (4-22)

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PRINCIPAL NOTATION

The principal symbols are defined. In defining derivatives the letters in parentheses are used to complete the symbols. The underlined letters denote the variables to which the derivatives are applied. The input and output parameters used in the computer program are defined on pages 56-60.

A	The program symbol for θ. In the input and output A is the direction in degrees from which the wave packet comes with respect to true north. Internally in the program A is the direction in radians with which the wave packet moves with respect to the positive x-axis.
AA	The wave packet direction at the new ray point.
AAA, AAAV	The average of the values at the new and previous ray points, respectively, of the wave packet and wavelet directions.
ABAR	The average of the wave packet directions at the present and new points.
AGR	The program symbol for g.
AK(i)	The program symbol for K_i where i = 1, 2, ..., 9.
AKFC	The program symbol for K_F.
AKR	The program symbol for K_R.
AKS	The program symbol for K_S.
AL(i)	The program symbol for L_i where i = 1, 2,...,9.
ALFA	The program symbol for α.
AMM, ANN	The maximum values of x and y, respectively, for a water depth grid.
ARAY	The program symbol for ρ. The directions of ARAY are defined following the same conventions used in the definitions of A.
AV	The program symbol for γ. The directions of AV are defined following the same conventions used in the definitions of A.
AVP	The program symbol for γ' or γ''.
AX, AY	The arrays used to store the locations of contour values, ray points, and tick marks.

BDZ	The program symbol for $d\beta/dt$.
BDZ5	The fifth order Runge-Kutta solution of $d\beta/dt$.
BZ	The program symbol for β.
BZTOL	The limiting value for $\lvert EBZ \rvert$ and $\lvert EBDZ \rvert$ in the Runge-Kutta calculations of β and $d\beta/dt$. If $\lvert EBZ \rvert$ or $\lvert EBDZ \rvert$ exceeds or is equal to BZTOL the time step interval is halved.
BZ5	The fifth order Runge-Kutta solution of β.
C	An array of 12 water depths from CMAT used to fit a quadratic surface in the vicinity of the ray point.
CALFA	The program symbol for α_c.
CDAD_	(X,Y) The program symbols for derivatives of θ in the x"y"-coordinate system (currents).
CDAVD_	(X,Y) The program symbols for derivatives of γ in the x"y"-coordinate system (currents).
CDGD_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of G in the x"y"-coordinate system (currents).
CDPHD_	(X,Y) The program symbols for derivatives of ϕ in the x"y"-coordinate system (currents).
CDUD_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of U in the x"y"-coordinate system (currents).
CDVD_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of v in the x"y"-coordinate system (currents).
CF	The program symbol for c_f.
CIN	If CIN is not zero, its value is the travel time between tick marks along a ray. In the input CIN is in seconds, but for the calculations CIN is converted to hours. If CIN is zero there are no tick marks on a ray.
CMAT	The water depth grid in a two dimensional array.
CNVRSA	The direction of the positive x-axis of the water depth and current grids with respect to true north. The use of this conversion angle permits the wave and current directions to be defined with respect to true north in the input and output.
CONTURC	An array containing the current profile values in feet/second or meters/second. There can be as many as 9 values.

CONTURD — An array containing the sounding water depths in feet or meters. There can be as many as 9 values.

CURR — The current magnitude in a two dimensional array.

CURX — The x-component current in a two dimensional array.

CURY — The y-component current in a two dimensional array.

CX — An array of 12 x-component current values from CURX used to fit a quadratic surface in the vicinity of the ray point.

CY — An array of 12 y-component current values from CURY used to fit a quadratic surface in the vicinity of the ray point.

c_f — The friction factor.

D — The program symbol for D_n.

DADX — The program symbol for the derivative of α in the x'y'-coordinate system (water depths).

DAVDX — The program symbol for the derivative of γ in the x'y'-coordinate system (water depths).

DELA — The change in the wave packet direction from the present to the new ray point.

DELAV — The change in the wavelet direction from the present to the new ray point.

DELX, DELY — The change in the values from the present to the new ray points of the x and y coordinates, respectively.

DEP — The program symbol for h.

DGD_ _ — (X,XX,XY,YY) The program symbols for derivatives of G in the x'y'-coordinate system (water depths).

DHD_ _ — (X,XX,XY,YY) The program symbols for derivatives of h in the x'y'-coordinate system (water depths).

D_n — The incremental distance in grid units between ray points.

DPHIDX — The program symbol for the derivative of ϕ in the x'y'-coordinate system (water depths).

DUD — The ratio of the phase speed at the present ray point to the value at the previous ray point.

DUD_ _	(X,XX,XY,YY) The program symbols for derivatives of U in the x'y'-coordinate system (water depths).
DVD_ _	(X,XX,XY,YY) The program symbols for derivatives of v in the x'y'-coordinate system (water depths).
DY	The number of grid units per inch or centimenter for a particular plot.
DZDD_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of ε in the x"y"-coordinate system (currents).
DZD_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of u in the x"y"-coordinate system (currents)
E	An array of 6 coefficients of the quadratic surface equation which is fitted to the 12 water depths in the array C.
EBDZ	The program symbol for $\varepsilon_{\tilde{\varepsilon}t}$.
EBZ	The program symbol for $\varepsilon_{\tilde{\varepsilon}}$.
EM	A two dimensional array of numbers used in computing the array E.
EX	An array of 6 coefficients of the quadratic surface equation which is fitted to the 12 x-component current values in the array CX.
EY	An array of 6 coefficients of the quadratic surface equation which is fitted to the 12 y-component current values in the array CY.
$\mathcal{E}$	The wave energy per unit area.
$\hat{e}_k$	A unit vector in the direction of γ.
$\hat{e}_m$	A unit vector in the direction of θ.
FK	The program symbol for κ_G. It is measureed in radians/grid unit.
FKAC	The program symbol for the packet ray curvature considering only variations due to currents.
FKAD	The program symbol for the packet ray curvature considering only variations due to water depths.
FKBAR	The average of the packet ray curvature at the present and new ray points.
FKK	The value of κ_G in radians/grid unit at the new ray point.

F	The average rate of energy transmission of the waves.
f	The frequency of the wave (1/T) relative to the current.
G	The geometric group speed.
GR	The program symbol for G_R.
G_R	The ray speed.
GRID	The number of feet or meters per grid unit for a particular water depth grid.
GT	The program symbol for G_T.
G_T	The speed of the advected group front in a current.
GTZERO	The value of G_T at the first ray point.
GZERO	The value of G at the first ray point.
g	The acceleration due to gravity.
H	The wave height.
HGT	The program symbol for H.
HGTZ	The initial value of H.
H_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of h in the xy-coordinate system (water depths).
h	The water depth.
IBDZ	Is set equal to zero in HEIGHT at the beginning of a ray. If a reflected ray is continued beyond a reflection point, IBDZ is set equal to one in HEIGHT when the calculations of β and dβ/dt are to resume. As a result, dβ/dt is determined analytically to restart the Runge Kutta calculations.
IDBDT	Is set equal to zero in SURFCE at the beginning of a ray. When a ray enters a current IDBDT is set equal to one in SURFCE. As a result, the initial value of dβ/dt for currents is determined analytically. This value is used in the Runge Kutta calculations of β and dβ/dt.
IFLG	When IFLG is zero a check is made in HEIGHT to determine if there should be a breakup of the time step interval in order to maintain the desired accuracy in the calculations of either β or the ray path. If there is a division of the time step interval, IFLG is set equal to one once the time step interval is sufficiently reduced. When IFLG

equals one further checks for a breakup of the time step interval are not made at new ray points until the breakup ends and calculations are resumed with the initial time step.

IHGT	If IHGT is set equal to zero the wavelet direction is determined in SURFCE. If IHGT is set equal to one the values of p and q are determined in SURFCE.
INUM	An index to count the ray points within the broken interval when there is a division of the initial time step interval.
IONCE	Is set equal to one in SURFCE at the beginning of a ray. IONCE is set equal to zero after the initial value of $d\beta/dt$ for currents is determined analytically in SURFCE. As a result, this calculation in not repeated for the ray.
IWAVIT	The flag IWAVIT is used when both water depth and current grids exist. Its purpose is to require accuracy in the calculation of the wavelet direction when iterating to a new ray point. At the beginning of the iteration process IWAVIT is set equal to zero. When successive wavelet estimates differ by less than a predetermined amount, IWAVIT is set equal to one. Iterations stop when both IWAVIT equals one and the ray curvature calculations have converged.
KCIN	The number of tick marks along a ray which do not coincide with ray points.
K_F	The friction coefficient.
K_i	Expressions used in the Runge-Kutta calculations of β and $d\beta/dt$ where $i = 1, 2, ..., 9$.
KMAX	The same as MAX except in DRAW where it is the sum of MAX and KCIN.
K_R	The refraction coefficient.
KREST	The number of tick marks along a ray.
K_S	The shoaling coefficient.
k	The wave number $2\pi/\lambda$.
LI	The number of lines of printout between page and column headings.
L_i	Expressions used in the Runge-Kutta calculations of β and $d\beta/dt$ where $i = 1, 2, ..., 9$.
ℓ	The perpendiculae distance between rays.

MAXQ	An index to number points along a ray at time intervals equal to the initial time step.
MIT	If MIT is 1 the wave packet curvature approximations in MOVE converge to one value. If MIT is 2 the curvature approximations converge to two values. If the curvature approximations do not converge and there is no reflection MIT is 3. If MIT is 4 a caustic or focal point is computed in HEIGHT. MIT is 5 if it is determined in HEIGHT that the wave breaks. When there is a reflection but the ray is not continued MIT is 6. If MIT is 7 more than one reflecton from the same point is determined in MOVE. If MIT is 8 the breakup time step determined in HEIGHT is less than 0.5 seconds.
MMAX	The dimension of the AX and AY arrays.
N	The ray number.
NBRKUP	Is zero except during a breakup of the time step interval when the value is one. After returning to MOVE from HEIGHT, the value of NBRKUP determines where the program resumes.
NDP	The water depth is determined in SURFCE. If the value is greater than 0, NDP is 1 (initialized in RAYN). If the water depth equals or is less than 0, NDP is 2.
NFK	The value of NFK is determined in SURFCE. If the ratio of the water depth to the deep water wavelength is greater than 0.64, NFK is 1. Otherwise, NFK is 2.
NFLAGR	Is set equal to zero in WAVPAK at the beginning of a ray. The value is changed to one in MOVE if there is a reflection and the ray is continued. If NFLAGR is one, in HEIGHT the value of NROPT is set equal to zero.
NFLECT	In WAVPAK, NFLECT is set equal to zero at the beginning of a ray. In HEIGHT, NFLECT is set equal to one for those ray points where the conditions for being close to a reflection point are met.
NFRACT	In WAVPAK, NFRACT is set equal to zero at the beginning of a ray. In HEIGHT, NFRACT is set equal to one for those ray points where there is a breakup of the time step interval due to insufficient accuracy in the Runge-Kutta calculations of β and $d\beta/dt$.
NGO	The value of NGO is determined in MOVE. If a ray point lies within one and one half grid units of a grid boundary NGO is 2. Otherwise, NGO is 1. (Initialized in RAYN.)
NOLINE	An index used to determine when to write page and column headings depending upon the number of lines of printout.

NOREF	An index to count the number of reflections at a ray point.
NPLOT	The plot number.
NREF	The value of NREF is determined in MOVE and it denotes the kind of reflection. When there is reflection due to Snell's law with phase velocity NREF is 1. When reflection occurs because the packet curvature iteration is not converging NREF is 2. If there is reflection because the ray point is too near a reflection point NREF is 3.
NRFLBU	In WAVPAK, NRFLBU is set equal to zero at the beginning of a ray. In HEIGHT, NRFLBU is set equal to one for those ray points where the conditions for being close to a reflection point are met. This causes the statement "REFLECT" to appear in the output.
NRFRBU	In WAVPAK, NRFRBU is set equal to zero at the beginning of a ray. In HEIGHT, NRFRBU is set equal to one for those ray points where there is a breakup of the time step interval due to insufficient accuracy in the Runge-Kutta calculations of β and $d\beta/dt$. This causes the statement "BETA" to appear in the output.
NROPT	The initial value of NROPT is determined in the input data. If NROPT is zero a ray is not continued beyond a reflection point. If NROPT is not zero a ray is continued beyond a reflection point. After a reflection NROPT is set equal to zero so that a ray is not continued beyond a second reflection point if one should exist.
NTOREF	If there is total reflection due to the wavelets NTOREF is set equal to one. Otherwise, NTOREF is set equal to zero. The reflection test is made in SURFCE.
NUMT	The number of divisions when there is a breakup of the initial time step interval.
n	An index to number ray points.
OMEGA	The program symbol for ω.
OMEG_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of ω in the x"y"-coordinate system (currents).
PATI	A program symbol for p. The value of p at the point prior to the new ray point.
PHI	The program symbol for Φ.
POT	A program symbol for p. The value of p at the new ray point.

POTC	The program symbol for p at the new ray point considering only variations due to currents.
POTD	The program symbol for p at the new ray point considering only variations due to water depths.
PREV	The value of v at the previous ray point.
PREVT	The value of v_T at the previous ray point.
P(i)	Program symbols for p. The values of p at points intermediate to the new and previous ray points where i = 1, 2, ..., 5.
P	A coeffiecient of the ray separation equation.
QATI	A program symbol for q. The value of q at the point prior to the new ray point.
QOT	A program symbol for q. The value of q at the new ray point.
QOTC	The program symbol for q at the new ray point considering only variations due to currents.
QOTD	The program symbol for q at the new ray point considering only variations due to water depths.
Q(i)	Program symbols for q. The values of q at points intermediate to the new and previous ray points where i = 1, 2, ..., 5.
q	A coefficient of the ray separation equation.
RT	The length of the x-axis in inches or centimeters for a given plot.
S	A two dimensional array of numbers used in computing the arrays E, EX, and EY.
SCL	The scale of the plot.
S_G	The arc length of a wave packet trajectory.
s_{ray}	The arc length of a ray.
s_v	The arc length of a monochromatic ray.
T	The wave period relative to the current.
TIMEQ	The travel time along a ray.

TT	The program symbol for the input wave period.
TTT	The program symbol for the Doppler shifted wave period.
t	Time.
U	The conventional group speed $d\omega/dk$.
UK	The program symbol for u_k.
UK_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of u_k in the x"y"-coordinate system (currents).
UM_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of u_m in the x"y"-coordinate system (currents).
u	The speed of the current.
u_k	The component of the current in the direction of [illegible], i.e., perpendicular to the wavelet front.
u_m	The component of the current in the direction of [illegible], i.e., perpendicular to the group front.
V	The program symbol for v.
VT	The program symbol for v_T.
v	The phase speed of a monochromatic wave.
v_T	The speed of the advected wavelet front in a current.
WAVNO	The program symbol for k.
WAVNO_	(X,Y) The program symbols for derivatives of k in the x"y"-coordinate system (currents).
WL	The program symbol for the deep water value of λ.
X	The program symbol for x.
XX	The program symbol for x at the rew ray point.
x	A Cartesian coordinate of the water depth grid.
x'	A Cartesian coordinate in a system chosen such that $\partial h/\partial y' = 0$.
x"	A Cartesian coordinate in a system chosen such that $\partial u/\partial y'' = 0$.
Y	The program symbol for y.

YVW	A one dimensional array used in computing the arrays E, EX, and EY.
YY	The program symbol for y at the new ray point.
y	A Cartesian coordinate of the water depth grid.
y'	A Cartesian coordinate in a system chosen such that $\partial h/\partial y' = 0$.
y''	A Cartesian coordinate in a system chosen such that $\partial u/\partial y'' = 0$.
Z	The program symbol for u.
ZD	The program symbol for ε.
ZD_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of ε in the xy-coordinate system.
ZT_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of u in in the xy-coordinate system.
ZX	The program symbol for the x-component of the current in the xy-coordinate system.
ZX_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of ZX in the xy-coordinate system.
ZY	The program symbol for the y-component of the current in the xy-coordinate system.
ZY_ _	(X,Y,XX,XY,YY) The program symbols for derivatives of ZY in the xy-coordinate system.
α	The angle the x'-axis is rotated with respect to the x-axis such that $\partial h/\partial y' = 0$.
α_c	The angle the x''-axis is rotated with respect to the x-axis such that $\partial u/\partial y'' = 0$.
β	The ray separation factor.
γ	The wavelet direction defined with respect to the positive x-axis.
Δt	The time step interval between ray points.
ε	The current direction with respect to the positive x-axis.
ε_β	The difference between the fourth and fifth order Runge-Kutta solutions of β.

$\varepsilon_{\beta t}$	The difference between the fourth and fifth order Runge-Kutta solutions of $d\beta/dt$.
θ	The wave packet direction defined with respect to the positive x-axis.
κ_G	The ray curvature of the wave packet.
κ_V	The ray curvature of a monochromatic wave.
λ	The wavelength.
π	3.1415927
σ	The ray direction defined with respect to the positive x-axis.
τ	The tangential stress per unit area at the bottom.
ϕ	The angle ($\theta-\gamma$).
ω	The radian frequency ($2\pi f$) of the wave relative to the current.

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Ray Curvature	Shoaling Coefficient	Water Depth
Wave Packet	Refraction Coefficient	Current
Total Reflection	Friction Coefficient	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The theory and numerical methods are presented for determining the paths and wave heights of gravity water wave packets. Both variable water depths and currents are considered. The wave height is computed accounting for the effects of shoaling, refraction, and energy dissipation. A ray curvature expression is used to determine the wave packet trajectories where the speed of the packet is given by $G = (d\omega/dk) \cos \alpha$. The symbol ω denotes the angular

frequency, k is the wave number, and ϕ is the difference between the direction of the wave packet and the direction of the wavelets within the packet. The wavelet direction is determined at each point of the wave packet trajectory. The wave packet and ray directions differ when there are currents. The calculations for variations in water depth are greatly simplified by choosing a coordinate system at each ray point in which one axis is aligned parallel with the direction of the gradient of the water depth. In a similar fashion, the calculations involving variations in current are simplified by choosing a coordinate system at each ray point in which an axis is taken parallel with the direction of the gradient of the current magnitude. Example printouts and plots are presented to illustrate the wave prediction method.

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DTIC

D_n	The incremental distance in grid units between ray points.
DPHIDX	The program symbol for the derivative of ϕ in the x'y'-coordinate system (water depths).
DUD	The ratio of the phase speed at the present ray point to the value at the previous ray point.

FKAC	The program symbol for the packet ray curvature considering only variations due to currents.
FKAD	The program symbol for the packet ray curvature considering only variations due to water depths.
FKBAR	The average of the packet ray curvature at the present and new ray points.
FKK	The value of κ_G in radians/grid unit at the new ray point.

SURFCE. As a result, the initial value of $d\beta/dt$ for currents is determined analytically. This value is used in the Runge Kutta calculations of β and $d\beta/dt$.

IFLG	When IFLG is zero a check is made in HEIGHT to determine if there should be a breakup of the time step interval in order to maintain the desired accuracy in the calculations of either β or the ray path. If there is a division of the time step interval, IFLG is set equal to one once the time step interval is sufficiently reduced. When IFLG

k The wave number $2\pi/\lambda$.

LI The number of lines of printout between page and column headings.

L_i Expressions used in the Runge-Kutta calculations of β and $d\beta/dt$ where $i = 1, 2, \ldots, 9$.

ℓ The perpendiculae distance between rays.

NGO	The value of NGO is determined in MOVE. If a ray point lies within one and one half grid units of a grid boundary NGO is 2. Otherwise, NGO is 1. (Initialized in RAYN.)
NOLINE	An index used to determine when to write page and column headings depending upon the number of lines of printout.

PATI	A program symbol for p. The value of p at the point prior to the new ray point.
PHI	The program symbol for Φ.
POT	A program symbol for p. The value of p at the new ray point.